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TOOLS FOR GEO-ECOLOGICAL MONITORING OF THE RESTORATION OF THE LAND AND PROPERTY COMPLEX OF TRANSPORT INFRASTRUCTURE

Mamonov K.
D. Sc. (Econ.), Prof., head of department,
department of land administration and geographic information systems
O.M. Beketov National University of Urban Economy in Kharkiv
http://orcid.org/0000-0002-0797-2609
Goi V.
D. Sc. (Econ.), senior lecturer,
department of land administration and geoinformation systems
O.M. Beketov National University of Urban Economy in Kharkiv
https://orcid.org/0000-0003-1822-4478
Shterndok E.
Cand. Sc. (Tech.), associate professor,
department of land administration and geoinformation systems
O.M. Beketov National University of Urban Economy in Kharkiv
https://orcid.org/0000-0003-1107-7401
Frolov V.
Cand. Sc. (Tech.), assistant,
department of land administration and geoinformation systems
O.M. Beketov National University of Urban Economy in Kharkiv
https://orcid.org/0000-0001-8045-3963
DOI: 10.5281/zenodo.21914065

Abstract

This article examines contemporary approaches and tools for geo-ecological monitoring of the reconstruction of the land and property complex of transport infrastructure in the context of post-war reconstruction in Ukraine. It justifies the need to establish a comprehensive monitoring system aimed at ensuring the effective management of transport facility reconstruction processes, restoring the functional condition of land resources and improving the environmental safety of territories. It has been established that the destruction of transport infrastructure as a result of military operations has led to significant changes in land-use patterns, a deterioration in the environmental condition of territories and a loss of the functional characteristics of land and property complexes, which necessitates the introduction of modern tools for spatial analysis and monitoring.

The paper examines the specific features of applying geoinformation systems, Earth remote sensing technologies, satellite monitoring, unmanned aerial vehicles, as well as intelligent analytical platforms based on artificial intelligence and machine learning technologies. It has been established that the use of these tools enables the rapid collection and processing of geospatial data, the assessment of the extent of damage to land and transport facilities, the identification of environmental risks, the monitoring of the implementation of restoration measures, and the forecasting of future territorial development.

A conceptual model for geo-ecological monitoring of the restoration of the land and property complex of transport infrastructure is proposed, comprising modules for spatial data collection, integration of geoinformation resources, environmental analysis, assessment of the land and property complex, intelligent forecasting and support for management decision-making. It has been demonstrated that the integrated use of modern digital technologies creates an information and analytical environment for making informed management decisions, enhances the efficiency of land resource utilisation and contributes to the implementation of sustainable development principles during the reconstruction of transport infrastructure.

Keywords: geo-ecological monitoring, transport infrastructure, land and property sector, geoinformation systems, remote sensing, territorial regeneration, sustainable development.

Introduction. Transport infrastructure is one of the fundamental elements underpinning the functioning of territorial systems and the socio-economic development of the state. As a result of hostilities, a significant number of transport infrastructure facilities in Ukraine have suffered substantial damage, leading to the disruption of transport links, land degradation and the loss of functional characteristics of land and property complexes.

The effective reconstruction of transport infrastructure requires the implementation of modern approaches to monitoring the condition of land, engineering structures, environmental parameters and the spatial characteristics of territories. Geo-ecological monitoring tools are of particular importance, as they enable comprehensive monitoring of the restoration processes of land and property complexes within the transport infrastructure and facilitate the making of informed management decisions. **Literature Re-**

view. Issues relating to the monitoring of land resources, the development of transport infrastructure and the use of geoinformation technologies are widely covered in the works of both domestic and international researchers. Considerable attention is paid to issues of spatial analysis, environmental assessment of territories, monitoring of land-use changes and the implementation of intelligent management systems for infrastructure facilities.

It should be noted that the theoretical principles for carrying out monitoring procedures, taking into account land and environmental parameters, are presented in the following studies [1–3].

At the same time, there remains a lack of research into the integration of geo-ecological, land-use, property and infrastructure data into a single decision-support system for the reconstruction of transport infrastructure, utilising monitoring procedures.

The aim of the study is to justify the use of modern geo-ecological monitoring tools for the reconstruction of the land and property complex of transport infrastructure and to establish the conceptual foundations for their integrated application.

Objectives of the study:

1. To analyse the current state of reconstruction of the land and property complex of transport infrastructure in the context of post-war recovery in Ukraine.
2. To investigate the theoretical and practical aspects of applying geo-ecological monitoring to ensure the effective management of transport infrastructure reconstruction processes.
3. To identify the main geo-ecological monitoring tools used to assess the condition of land, transport facilities and engineering structures.
4. To highlight the potential of geoinformation systems in providing information and analytical support for the reconstruction of the land and property complex of transport infrastructure.
5. To justify the specific features of using Earth remote sensing and satellite monitoring technologies to monitor changes in the condition of territories and transport facilities.
6. To identify areas of application for unmanned aerial vehicles within the system of geo-ecological monitoring of reconstruction processes.
7. To investigate the potential for using intelligent analytical platforms, artificial intelligence and machine learning technologies for processing geospatial data and forecasting reconstruction outcomes.
8. To develop a conceptual model for geo-ecological monitoring of the restoration of the land and property complex of transport infrastructure, based on the integration of spatial, environmental, technical and management data.
9. To justify approaches for improving the effectiveness of management decision-making regarding the restoration of transport infrastructure, based on the results of geo-ecological monitoring.

Main section. Geo-ecological monitoring is a comprehensive system for the observation, collection, processing and analysis of spatial, environmental and technical data concerning the condition of territories and infrastructure facilities. In the context of the reconstruction of the land and property complex of transport infrastructure, geo-ecological monitoring is aimed at:

– assessing the extent of damage to land and transport facilities;
– identifying environmental risks and threats;
– monitoring infrastructure restoration processes;
– assessing the efficiency of land resource use;
– forecasting the consequences of reconstruction measures;
– supporting management decision-making.

One of the key tools of geo-ecological monitoring is geographic information systems (GIS), which enable the integration of diverse spatial data within a single digital environment.

GIS enable:

– the creation of digital maps of damage to transport infrastructure;
– the analysis of the spatial distribution of environmental risks;
– the modelling of reconstruction scenarios;
– the development of interactive geoportals to support management decision-making;
– the multi-criteria assessment of territories.

The use of modern platforms such as ArcGIS, QGIS and web-based geoinformation services ensures that information is updated promptly and is accessible to public authorities, investors and the general public.

An important element of geo-ecological monitoring is the use of satellite remote sensing data.

Satellite monitoring enables:

– the detection of damage to road and rail infrastructure;
– the assessment of changes in land cover;
– the monitoring of erosion and land degradation processes;
– the monitoring of flooding and landslide processes;
– the determination of the progress of transport infrastructure reconstruction.

Data from the Sentinel and Landsat satellite systems are of particular importance, as they provide regular updates on the condition of territories. The use of unmanned aerial vehicles (UAVs) significantly improves the accuracy of geo-ecological monitoring. The main advantages of using UAVs are:

– obtaining high-precision orthophotos;
– creating digital elevation models;
– prompt detection of damage to infrastructure;
– monitoring the progress of reconstruction work;
– monitoring hazardous areas.

The use of UAVs enables the collection of spatial data with centimetre-level accuracy and significantly reduces the costs of field surveys.

Current developments in digital technologies are facilitating the integration of artificial intelligence and machine learning systems into geo-ecological monitoring processes.

Intelligent systems provide:

– automatic classification of damaged areas;
– forecasting of environmental risks;
– analysis of large datasets of geospatial data;

– assessment of the effectiveness of restoration projects;

– formulation of recommendations for optimising the use of land resources.

The use of artificial intelligence significantly improves the speed and accuracy of management decision-making.

The proposed model for geo-ecological monitoring of the reconstruction of the land and property complex of transport infrastructure comprises the following functional modules:

1. Spatial data collection module.
2. Geoinformation resource integration module.
3. Environmental analysis module.
4. Land and property complex assessment module.
5. Intelligent forecasting module.
6. Management decision support module.

The integration of these modules enables the creation of a digital environment for managing transport infrastructure reconstruction processes and improves the efficiency of land resource utilisation.

Conclusions. The study found that the effectiveness of the restoration of the land and property complex of transport infrastructure depends to a large extent on the implementation of modern geo-ecological monitoring tools. The most promising approaches are integrated solutions combining geoinformation systems, Earth remote sensing technologies, unmanned aerial vehicles and intelligent analytical platforms.

The integrated application of these tools ensures greater reliability in assessing the condition of land and property complexes, enables prompt decision-making and facilitates the development of scientifically sound strategies for the reconstruction of transport infrastructure.

It has been established that geo-ecological monitoring is a key element in managing the processes of reconstructing the land and property complex of transport infrastructure. The use of geoinformation systems, Earth remote sensing technologies, unmanned aerial vehicles and intelligent data analysis systems enables a comprehensive assessment of the condition of territories, monitoring of the implementation of reconstruction measures and forecasting the future development of transport infrastructure.

Further research should focus on developing an intelligent geoinformation system for geo-ecological monitoring of the reconstruction of transport infrastructure land and property complexes, which will ensure the integration of spatial, environmental, economic and management data within a single digital environment.

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GEOTECHNICS

GEOMECHANICS IN MINING: FUNDAMENTAL PRINCIPLES, RESEARCH METHODS AND ENGINEERING SOLUTIONS

Sultanova G.
Scientific Researcher, Institute of Geology, Ministry of Science and Education of the Republic of Azerbaijan
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Abstract

This article systematizes modern scientific concepts of geomechanics as a fundamental discipline ensuring the safety and efficiency of mining operations. The physical and mechanical properties of rocks, patterns of stress-strain state formation in rock masses, major classification systems (RMR, Q, MRMR, SRMR), field and laboratory research methods, as well as numerical modeling approaches are considered. The results of comprehensive studies at various types of deposits are presented: wollastonite and calcite quarries, perlite mines, copper and iron ore sites. Special attention is paid to weak rocks with properties analogous to some formations of Azerbaijan, for which empirical nomograms linking the rock class by SRMR, slope height, and safety factor have been developed. For the first time, data on geomechanical support for the development of Azerbaijani deposits (Dashkesan, Gadabay, Kelbajar, Zangilan, Karabakh) are presented. Practical recommendations for the selection of support systems, drainage, and optimization of slope angles for the conditions of the Azerbaijani region are substantiated. It is shown that the combination of numerical methods with instrumental monitoring allows achieving prediction accuracy with an error of no more than 10–15%.

Keywords: rock mechanics, rock mass classification, RMR, Q-system, SRMR, weak rocks, slope stability, underground mining, finite element method, discrete element method, limit equilibrium, rock support, stress-strain state, Hoek-Brown criterion, shear strength, cohesion, friction angle, safety factor, pillar stability, rock bolting, shotcrete, hydrogeological factor, scale effect, digital twin.

Introduction. Geomechanics (rock mechanics) is a scientific discipline that studies the behavior of rock masses under the influence of natural and technogenic loads. This field of knowledge serves as the theoretical foundation for the design, construction, and operation of underground and open-pit mines, as well as civil structures interacting with the rock environment. The emergence of geomechanics as an independent science dates back to the mid-20th century, when the First International Congress on Rock Pressure was held in Liège in 1951, and the founding conference of the International Society for Rock Mechanics (ISRM) took place in Lisbon in 1966. In 2007, the ISRM published a fundamental collection of recommended methods, which became the international standard for testing, classification, and monitoring. In modern conditions, the mining industry faces a number of challenges: transition to deep horizons (over 1000 m), expansion of open-pit scales, development of deposits in complex structural and hydrogeological conditions, as well as stricter environmental and industrial safety requirements. Traditional empirical approaches based on engineering experience and analogies often prove insufficient for complex objects. They are being replaced by numerical modeling methods that allow accounting for anisotropy, heterogeneity, nonlinear behavior, and the presence of structural disturbances. However, the success of numerical calculations directly depends on the reliability of initial data obtained during field and laboratory studies. Therefore, the key task of modern geomechanics lies in the integrated use of all available methods — from field observations to computer modeling — to obtain reliable predictions and substantiated design solutions. This work presents a comprehensive review of theoretical foundations, research methods, and practical applications of geomechanics using the example of several deposits of various types, including the complex conditions of weak rocks. Special attention is paid to quantitative calculations and numerical examples illustrating the application of theoretical principles in engineering practice.

Theoretical Foundations of Geomechanics with Mathematical Examples

A rock mass is a heterogeneous medium consisting of intact blocks and systems of structural discontinuities (fractures, bedding, faults). Its mechanical behavior is determined both by the properties of the blocks themselves (strength, elasticity, plasticity, creep) and by the characteristics of discontinuities (orientation, frequency, aperture, roughness, infill). The stress state of the mass is formed under the action of gravitational forces (weight of overlying rocks) and tectonic stresses that can persist in rocks for millions of years. The vertical stress component at depth z is calculated by the simple formula $\sigma_v = \gamma z$, where γ is the unit weight of the rock, which for most rock types is about 0.027 MN/m^3. Let us perform a numerical example: for depth $z = 500$ m, the vertical stress will be $\sigma_v = 0.027 \times 500 = 13.5$ MPa. Horizontal stresses often exceed vertical ones, especially at shallow depths. To estimate the ratio of horizontal σ_h to vertical stress, the coefficient of lateral pressure $k = \sigma_h / \sigma_v$ is used, which according to the empirical Sheorey relationship can be estimated as $k = 0.25 + 7E_h(0.001 + 1/z)$, where E_h is the deformation modulus in the horizontal direction, GPa. Let us continue the numerical example. Let $E_h = 10$ GPa, $z = 500$ m. Then $0.001 + 1/500 = 0.001 + 0.002 = 0.003$. Next, $7 \times 10 = 70$. The product $70 \times 0.003 = 0.21$. Consequently, $k = 0.25 + 0.21 = 0.46$. This means that the horizontal stress is only 46% of the vertical stress, which is typical for areas with relatively calm tectonics. However, in zones of active tectonic stresses, k can reach values of 3–5 at depths up to 100 m, as confirmed by measurements at mines. In the presence of two principal horizontal stresses σ_{h1} and σ_{h2}, their average value is used in stability calculations.

The strength properties of intact rock are characterized by the uniaxial compressive strength (σ_c), tensile strength (σ_t, usually 5–15% of σ_c), as well as shear strength parameters — cohesion (c) and angle of internal friction (ϕ), which are related by the Coulomb-Mohr equation: $\tau = c + \sigma_n \tan\phi$. These parameters are determined in laboratory tests; however, their values for the mass as a whole can be an order of magnitude or more lower due to the scale effect. To account for fracturing, semi-empirical strength criteria are used, such as the Hoek-Brown criterion, which in general form is written as $\sigma_1 = \sigma_3 + \sigma_c(m\sigma_3/\sigma_c + s)^a$, where m, s, a are empirical constants depending on the mass quality. For intact rock, $s = 1$, $a = 0.5$; for highly fractured masses, s approaches zero. An important place is occupied by rheological properties, in particular creep, which manifests itself in three stages: decaying, steady-state, and progressive. Creep is critical for assessing the long-term stability of pillars and underground structures. To describe creep, the Burgers model is often used, which is a series connection of Kelvin and Maxwell elements. The creep equation at constant stress σ_0 has the form:

$$\varepsilon(t) = \sigma_0/E_M + \sigma_0/E_K\left(1 - \exp(-E_K t/\eta_K)\right) + \sigma_0 t/\eta_M,$$

where E_M is the Maxwell elastic modulus, E_K and η_K are the Kelvin modulus and viscosity, η_M is the Maxwell viscosity. Water has a multifaceted effect: through pore pressure it reduces effective stresses, decreases cohesion in clayey interlayers, causes hydrodynamic pressure during filtration, and can lead to rock weakening. Effective stresses are determined by Terzaghi's principle: $\sigma' = \sigma - u$, where u is pore pressure. During water filtration through a porous medium, a hydrodynamic force arises $F = \gamma_w iV$, where γ_w is the unit weight of water, i is the hydraulic gradient, V is the volume of rock. Thus, the hydrogeological factor often becomes decisive for the stability of slopes and underground excavations, especially during periods of intense precipitation. To assess the influence of water on slope stability, a calculation is performed taking into account filtration forces, which allows adjusting the slope angle or designing drainage systems.

Classification Systems of Rock Masses with Calculation Examples

Rock mass classifications serve as a bridge between detailed geological description and engineering calculations. They allow quantitative assessment of mass quality, selection of support type and parameters, determination of the stable span of an excavation or slope angle. The most common in world practice are the RMR, Q, MRMR, and SRMR systems. The RMR system, developed by Bieniawski, is based on six parameters: rock strength, RQD index, spacing of discontinuities, condition of discontinuities, groundwater conditions, and orientation. Each parameter is assessed in points, the sum of which gives RMR. For example, rock with strength $\sigma_c = 100$ MPa receives 7 points (on the scale), RQD = 80% — 17 points, spacing between fractures 200–600 mm — 10 points, fracture condition (slightly rough, with infill <1 mm) — 25 points, dry excavation — 15 points, favorable orientation — 0 points. The sum will be 7 + 17 + 10 + 25 + 15 + 0 = 74 points, which corresponds to class II (good rock). For each class, standard support solutions are proposed: for class II — roof bolts 3 m long with 2.5 m spacing and mesh if necessary. The Q system, proposed by Barton, uses a logarithmic scale and is calculated as the product of three ratios: $Q = (RQD/J_n) \cdot (J_r/J_a) \cdot (J_w/SRF)$, where J_n is the number of joint sets, J_r is roughness, J_a is alteration of joint walls, J_w is water reduction factor, SRF is stress reduction factor. Let us consider a specific example of Q calculation for a perlite mine mass. Let RQD = 70%, number of joint sets $J_n = 3$ (two main and one random), roughness $J_r = 1.5$ (flat, slightly rough walls), alteration $J_a = 0.75$ (hard, non-softening infill), water reduction factor $J_w = 1.0$ (dry excavation), stress reduction factor $SRF = 2.5$ (single shear zone in competent rock). Then $Q = (70/3) \cdot (1.5/0.75) \cdot (1.0/2.5) = 23.33 \cdot 2 \cdot 0.4 = 18.66$. The value $Q \approx 18.7$ corresponds to good rock quality, which is confirmed by the absence of need for heavy support. For approximate assessment of required support by Q, Barton's nomograms are used, where for $Q = 18.7$ and equivalent excavation size $D_e = 5$ m (at ESR = 1.6 for permanent mine excavations) bolting with 2–3 m spacing without shotcrete is recommended. For mining in high-stress conditions and blasting impacts, a modification of RMR has been developed — the MRMR system by Laubscher, which introduces correction factors for weathering, blast damage, and stress redistribution. For example, if RMR = 60, and the correction for blasting is –5 points, for weathering –3 points, for stress –2 points, then MRMR = 60 – 5 – 3 – 2 = 50 points. Based on this value, the support type is selected: for 50 points — bolting with shotcrete. Special attention deserves the SRMR system, created by Robertson specifically for weak rocks ($RMR < 40$), where standard classifications give overestimated strength values. SRMR is based on back-analysis of landslides and allows correct determination of parameters c and ϕ. For example, for class IVa (SRMR 35–40), cohesion is 0.61–0.70 MPa, friction angle 16–20°. For class Vb (SRMR 10–15), cohesion is only 0.06–0.26 MPa, friction angle 8–10°. In practice, it is recommended to apply two or three classifications simultaneously for cross-checking, and the choice of a specific system should be determined by the rock type and the engineering problem being solved.

Field and Laboratory Research Methods with Mathematical Data Processing

The reliability of geomechanical predictions is determined by the quality of initial data obtained during field and laboratory work. Field investigations include structural mapping of outcrops and core with recording of orientation, density, roughness, aperture, and infill of fractures. For these purposes, geological compasses, laser scanners, and photogrammetric methods are used. Processing of mapping data includes construction of stereonets and statistical analysis of fracture orientations. For example, using the DIPS program, average orientations of fracture sets, their dispersion, and density are calculated. Drilling with double and triple core barrels ensures high core recovery, from which RQD is determined and lithological description is performed. RQD is calculated as the sum of lengths of core pieces exceeding 100 mm divided by the total interval length, multiplied by 100%. Stress measurements in the mass are performed by three main methods: overcoring with strain gauge cells, hydraulic fracturing, and flat jack method. The first provides information about the full stress tensor at a point, the second about the magnitudes of principal stresses in the plane perpendicular to the borehole, the third about surface stresses on excavation walls. When processing overcoring data, formulas are used to calculate principal stresses from strains measured in three directions. For example, if strains $\varepsilon_1, \varepsilon_2, \varepsilon_3$ are known at angles 0°, 60°, 120°, then the maximum and minimum principal stresses are calculated by the formulas:

$$\sigma_1 = \frac{E}{6d}\left[(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + \frac{\sqrt{2}}{2}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}\right]$$

$$\sigma_2 = \frac{E}{6d}\left[(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) - \frac{\sqrt{2}}{2}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}\right]$$

where E is the elastic modulus, d is the borehole diameter. Hydrogeological observations include measurements of groundwater levels, water inflows, packer tests for permeability, and water sampling. From packer test data, the permeability coefficient is calculated by the formula $k = Q/(2\pi lh)$, where Q is water flow rate, l is interval length, h is head. Instrumental monitoring is carried out using multi-point extensometers, vibrating wire strain gauges, load cells, tape convergence meters, and inclinometers, which allow tracking displacements, deformations, and loads in real time. Monitoring data are statistically processed to identify trends and anomalies. For example, based on extensometer readings, a displacement vs. time graph is constructed, from which the deformation rate is determined and the time to reach the limit state is predicted.

Laboratory tests on samples taken from core or monoliths are conducted according to ISRM and ASTM standards. The main volume of tests is the determination of uniaxial compressive strength with recording of the complete stress-strain curve, which allows calculating the elastic modulus and Poisson's ratio. Tensile strength is determined by the Brazilian method (disc splitting) or bending test. Triaxial tests at various levels of confining pressure make it possible to construct the Mohr envelope and obtain values of c and ϕ. For this purpose, test results are plotted on a graph in coordinates σ — τ, a tangent is drawn through the points, the slope angle of which gives ϕ, and the intercept on the τ axis gives c. Direct shear along rock-rock or rock-concrete contact models the conditions of foundation bases and anchor work. In addition, creep tests, determination of weathering resistance index (slake-durability), permeability and physical properties (density, porosity, moisture content) are conducted. It is important to note that laboratory data should always be compared with field test results, as the scale effect can lead to discrepancies of 5–15 times. To correct laboratory data, empirical weakening coefficients are used that account for fracturing and weathering of the mass.

Numerical Modeling in Geomechanics with Problem-Solving Examples

Numerical modeling is a powerful tool for solving complex geomechanical problems that have no analytical solutions. It allows reproducing heterogeneous, anisotropic, nonlinear media, taking into account construction staging, changes in properties over time, and complex boundary conditions. The main groups of methods include: continuum modeling methods, discrete modeling methods, and limit equilibrium methods. The finite element method (FEM) is used for continuous media, where the mass is considered as a continuum with elastic, elastoplastic, or viscoplastic properties. Programs such as FLAC, ANSYS, and ABAQUS allow modeling pore pressure, thermal processes, and dynamic loads. When solving a slope stability problem by the finite element method, a mesh of elements is constructed, boundary conditions are set (fixation on the sides and bottom, self-weight loading), then stress and strain calculations are performed. Based on the results, zones of plastic deformation are identified and the safety factor is calculated by the strength reduction method (c and ϕ are reduced until loss of stability). FEM is effective in analyzing circular and complex sliding surfaces in weak rocks.

For fractured masses, where behavior is determined by block structure, the discrete element method (DEM) is applied. Programs UDEC (2D) and 3DEC allow modeling a set of rigid or deformable blocks interacting at contacts considering friction, cohesion, opening, and sliding. The method allows tracking large displacements, collapses, fallouts, and the performance of rock bolting. Input parameters include block properties (density, elastic modulus) and contact properties (normal and shear stiffness, friction angle, cohesion, tensile strength). The solution is performed in time using an explicit integration scheme of motion equations. The limit equilibrium method, implemented in GALENA and SLIDE programs, remains fundamental in engineering practice due to its simplicity and clarity. It is based on comparing driving and resisting forces along a presumed sliding surface and uses the Bishop, Spencer, Sarma, or Fellenius methods. For example, the Bishop method assumes a circular sliding surface and accounts for inter-slice forces, satisfying moment equilibrium conditions. An iterative procedure allows finding the minimum safety factor. The disadvantage is the lack of consideration of deformations before failure; however, for preliminary assessments and comparative analyses, it is indispensable. Recently, hybrid models have been developing, combining the advantages of different approaches, for example, FEM for the far field and DEM for the near zone around the excavation. This allows reducing computational costs and increasing accuracy in local zones with intense fracturing. An example is a model where FEM is used to calculate stresses in the remote region, and DEM is used to model the behavior of blocks near the excavation considering their possible displacement and fallout.

Stability Analysis of Slopes and Underground Excavations with Numerical Examples

The stability of slopes and underground excavations is a key problem in mining, as it affects the safety of personnel, preservation of equipment, and economic efficiency of development. The main types of slope failure include planar sliding along one weakness plane, wedge sliding along two intersecting planes, circular sliding in weak and highly fractured rocks, toppling of elongated blocks, and raveling of individual pieces. For each type, kinematic analysis methods using stereonets have been developed, which allow identifying potentially dangerous combinations of fracture and slope orientations. For example, for planar sliding, the activation condition is: the dip direction of the fracture must coincide with the dip direction of the slope, the dip angle of the fracture must be less than the slope angle but greater than the friction angle. Calculation of the safety factor F_s as the ratio of resisting forces to driving forces is the basis of quantitative assessment. Let us perform a numerical example for planar sliding. Suppose a block with weight $W = 1000$ kN lies on a slope. The contact area along the sliding plane is $A = 10$ m^2. The slope angle is $\psi = 35°$, the friction angle along the contact is $\phi = 30°$, cohesion $c = 50$ kPa (50 kN/m^2). We determine the normal and shear components of the weight: $N = W\cos\psi = 1000 \times 0.819 = 819$ kN, $T = W\sin\psi = 1000 \times 0.574 = 574$ kN. The friction force is $F_f = N\tan\phi = 819 \times 0.577 = 473$ kN. The cohesion force is $F_c = c \times A = 50 \times 10 = 500$ kN. The total resisting force $R = 473 + 500 = 973$ kN. The safety factor $F_s = R/T = 973/574 = 1.69$. A value above 1.5 indicates a stable condition. If cohesion were absent ($c = 0$), then $F_s = 473/574 = 0.82$, which would indicate instability. This example clearly demonstrates that even small cohesion (50 kPa) can drastically increase stability. When water is present in the fracture, pore pressure reduces the effective normal stress: $N' = N - U$, where U is the pore pressure force equal to $u \cdot A$. If $u = 100$ kPa, then $U = 1000$ kN, $N' = 819 - 1000 = -181$ kN (negative value means the block is floating), which leads to catastrophic reduction in stability.

For wedge sliding, the calculation is more complex and requires determining the resultant force along the line of intersection of two planes. In underground excavations, stability is ensured by support systems, the choice of which depends on the rock class by RMR or Q. For good rocks (RMR > 60, Q > 10), local bolting or no support is sufficient. For fair rocks (RMR 41–60, Q 4–10), systematic bolting with shotcrete is applied. In poor and very poor rocks (RMR < 40, Q < 4), heavy steel sets with 0.75–1.5 m spacing and full shotcreting are used. It is important that support is installed as close to the face as possible, before significant displacements begin, which allows maintaining the load-bearing capacity of the mass. For rock bolts, the load capacity is calculated by the formula: $P = \pi d l \tau$, where d is the bolt diameter, l is the anchorage length, τ is the bond strength at the bolt-rock interface. For example, with $d = 0.02$ m, $l = 2$ m, $\tau = 2$ MPa, we get $P = 3.14 \times 0.02 \times 2 \times 2 = 0.251$ MN, which corresponds to 25 tons.

Application of Geomechanics at Specific Deposits with Detailed Calculations

The practical significance of geomechanical studies is illustrated by the example of several sites. A wollastonite and calcite quarry was developed with slope angles of 28–33°, which led to losses of up to 17% of balance reserves. Conducted studies showed that the rocks are represented by skarns, pegmatites, and wollastonites with strength from 60 to 820 MPa. Classification by RMR gave 53–58 points (fair rock), by Q — 7.5–8.2. Stereonet analysis excluded planar and wedge sliding. Based on modeling in GALENA, it was proposed to increase the slope angle to 45–50° with the use of rock bolts and mesh. The safety factor was 1.2–1.3, which is acceptable under monitoring conditions. Implementation of the recommendations allowed extracting an additional more than 1 million tons of ore. At the weak iron ore mines of Goa (India), where rocks are represented by weathered laterites, clays, and altered dikes with very low strength, the use of RMR is impossible. Here the SRMR system was applied. Based on back-analysis of 50 landslides, nomograms were constructed linking SRMR, slope height, and angle. For class IVa (SRMR 35–40, $c = 0.61$–0.70 MPa, $\phi = 16$–$20°$) at height 35 m the angle should be 48°, at 90 m — 38°. For class Vb (SRMR 10–15, $c = 0.06$–0.26 MPa, $\phi = 8$–$10°$) respectively 37° and 27°. A drainage system (deep wells and surface water interception) and slope reinforcement with geogrids were proposed, which reduced the number of landslides by 70%. The perlite mine is composed of basalts, rhyolites, and perlite itself with high strength and RQD 60–80%. RMR = 50–56, Q = 10–21. For a room-and-pillar system with pillar width 4 m, height 6 m, and span 12 m, the safety factor is 1.3; at span 14 m it drops to 0.9. Let us calculate it formally: $\sigma_p = \gamma z\left(1 + W_o/W_p\right)$. With $\gamma = 0.2$ MN/m^3, $z = 50$ m, $W_o = 12$ m, $W_p = 4$ m we get $\sigma_p = 0.2 \cdot 50 \cdot (1 + 3) = 10 \cdot 4 = 40$ MPa. The pillar strength is $\sigma_{ksa} = 38.6$ MPa, so $F_s = 38.6/40 = 0.96$, which is below 1.0. This confirms the need to increase the pillar width to 5 m or reduce the span to 10 m. At a copper mine with depth 87 m, rocks are represented by epidiorites and schists. Measurements showed high horizontal stresses ($k = 2.86$–2.93). RMR = 53–58, Q = 5.3. The barrier pillar under the river was assessed as stable. For stopes with backfill, bolts and shotcrete were recommended; convergence monitoring showed displacements of no more than 15 mm. When calculating the pillar under the river, the additional load from the water column was taken into account, which amounted to $\sigma_w = \gamma_w H = 10 \times 30 = 300$ kPa = 0.3 MPa, which was included in the total stress.

Geomechanical Studies and Recommendations for Deposits of Azerbaijan

The mining industry of Azerbaijan has a centuries-old history and is currently actively developing, especially in connection with the liberation of the territories of Karabakh and Eastern Zangizur. The most significant ore districts are Dashkesan (iron ores), Gadabay (copper, gold), Kelbajar-Zangilan (gold, polymetals), and the Karabakh region (gold, copper, molybdenum).

The geological conditions of these regions are characterized by high fracturing, a developed weathering zone, tectonic disturbance, and seasonal waterlogging of rocks, which creates serious challenges for ensuring the stability of slopes and underground excavations.

For the Dashkesan iron ore deposit, composed of andesites, tuffs, and limestones, comprehensive geomechanical studies have been conducted. The strength of andesites is $\sigma_c = 120–180$ MPa, cohesion $c = 5–8$ MPa, angle of internal friction $\phi = 35°–42°$. According to structural mapping data, three joint systems have been identified: northeastern (predominant, dipping 50–70° to the southeast), sublatitudinal, and submeridional. RQD ranges from 55% to 78%, RMR is 56–62 points (class II–III), Q = 8–12. Stress measurements by overcoring at a depth of 200 m showed $\sigma_h = 12–16$ MPa, $\sigma_v = 5.4$ MPa, $k = 2.2–3.0$, which is explained by the tectonic activity of the Lesser Caucasus. Based on numerical modeling in FLAC and UDEC, it was recommended to increase the overall pit slope angle from 35° to 42° with bench height 12 m and the use of rock bolts (length 4 m, spacing 2 m) and drainage wells. The safety factor was 1.3–1.4. Implementation of the recommendations allowed increasing ore recovery by 8–10%.

For the Gadabay copper-gold deposit, mined underground by a room-and-pillar system, ore bodies with thickness 2–8 m at depths 150–400 m are characteristic. Host rocks are limestones and sandstones with $\sigma_c = 60–90$ MPa, $c = 3–5$ MPa, $\phi = 30°–35°$. RMR = 48–55, Q = 6–12. Stereonet analysis showed the presence of two dominant fracture systems dipping to the northwest. To ensure stope stability, bolts 3 m long with 1.8 m spacing and 80 mm shotcrete were recommended. Convergence monitoring showed displacements of 8–12 mm, which is within permissible values. In oxidation zones with SRMR = 25–30, flatter slope angles and enhanced drainage are applied.

The Kelbajar-Zangilan ore district is characterized by gold-polymetallic ores confined to zones of tectonic faults. The rocks are highly weathered, cohesion is 0.1–0.5 MPa, friction angle 10–20°, which corresponds to classes IV–V by SRMR. For quarry slopes at heights up to 50 m, angles of 30–35° are recommended with mandatory drainage and reinforcement with gabions and geogrids. For underground workings, combined support is proposed: bolts 2–3 m + 100 mm shotcrete + light sets if necessary. Dewatering is carried out by deep wells and drainage galleries.

For the Karabakh region, where significant reserves of gold, copper, and molybdenum have been explored, preliminary geomechanical assessments have been conducted. For hard rocks (granitoids, andesites), slope angles of 38–42° at heights up to 80 m are recommended, for weathered zones — 28–32° with drainage galleries. An instrumental monitoring program has been developed, including extensometers, inclinometers, and piezometers.

Based on the generalization of experience for all Azerbaijani deposits, regional coefficients for converting laboratory strength to mass strength are proposed: for andesites 0.4–0.6, for limestones 0.3–0.5, for weathered rocks 0.1–0.2. Minimum safety factors are recommended: for temporary workings 1.2–1.3, for permanent ones — 1.5–2.0, for critical structures (capital shafts, tailings dams) — no less than 1.5. Special attention is paid to seasonal waterlogging: during the monsoon period (May-September), drainage and monitoring should be intensified, and the pace of mining in risk zones should be reduced.

Discussion of Results, Uncertainties, and Practical Recommendations

A comparative analysis of classification systems confirms that RMR and Q give consistent results for medium and high strength masses, but for weak rocks ($RMR < 40$) a transition to SRMR is necessary, which is based on actual landslide data and provides more reliable estimates of c and ϕ. It is also important to note that the water factor is often decisive: water saturation can reduce F_s by 20–30%. Therefore, drainage is the most effective and economical way to increase stability. Numerical modeling should be performed with calibration against field data; in the absence of calibration, it is recommended to use conservative property estimates or vary parameters over a wide range. The prediction error with adequate calibration is 10–15%, which is acceptable for engineering practice. However, uncertainties related to the spatial variability of rock properties, measurement inaccuracies, and model simplifications should be taken into account. For quantitative uncertainty assessment, probabilistic methods such as the Monte Carlo method are used, where input parameters are specified as probability distributions and multiple calculations are performed to obtain the distribution of F_s. This allows determining the probability of failure, rather than just the deterministic value. Instrumental monitoring should be an integral part of any critical project. It allows not only verifying design solutions but also timely detecting anomalies, taking corrective measures, and preventing accidents. It is recommended to set threshold values of controlled parameters (displacements, loads, pressure) with subsequent automatic alarm when exceeded. For example, if the displacement rate exceeds 1 mm/day, it is necessary to intensify control and take stabilization measures. To achieve maximum efficiency, all types of data — field, laboratory, model, and monitoring — should be combined into a single information system that allows continuous analysis and adjustment of projects at all stages of development. The transition to digital twins of rock masses opens new opportunities for forecasting and risk management. A digital twin is a dynamic model updated in real time with monitoring data, which allows quickly assessing the state of the mass and predicting its behavior under various scenarios.

Conclusion and Development Prospects

The conducted research and generalization of world experience allow formulating the following conclusions. First, geomechanics is a fundamental discipline without which scientifically based design of mining excavations in modern conditions is impossible. Its role increases with increasing depth, complexity, and responsibility of objects. Second, the reliability of geomechanical calculations is ensured by the integrated application of field, laboratory, and numerical methods, with key importance being the calibration of models against field data. Third, the choice of classification system should be determined by the rock type: for weak masses, transition to SRMR is mandatory, and for hard and semi-hard rocks, RMR and Q are preferable. Fourth, the water factor is often decisive for stability, so drainage should be a priority measure. Fifth, the nomograms and recommendations for slope angles and support parameters developed at specific sites have proven their effectiveness and can be adapted for similar conditions. Finally, promising directions for further development are the creation of probabilistic risk assessment methods, development of digital twins of rock masses, and implementation of automated monitoring systems with feedback, which will allow the transition to intelligent management of mining operations based on real-time data. Of particular interest is the use of machine learning methods for predicting stability based on large datasets, as well as the development of new materials for support (composites, self-stressing bolts) and improvement of waterproofing methods. The implementation of these innovations will improve safety, reduce costs, and increase the completeness of mineral extraction.

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MATHEMATICS

ON A BOUNDARY VALUE PROBLEM FOR AN ODD-ORDER EQUATION WITH MULTIPLE CHARACTERISTICS

Xashimov A.
Tashkent State Economic University, Tashkent, Uzbekistan
Kurbanov O.
Tashkent State Economic University, Tashkent, Uzbekistan
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Abstract

This work investigates a nonlinear boundary value problem for an odd-order nonlinear differential equation with multiple characteristics in a curvilinear domain. We prove the unique solvability of this problem. To establish uniqueness, we employ the method of energy integrals in conjunction with elementary inequalities and a Friedrichs type inequality. For the existence proof, we introduce an auxiliary problem and construct its Green's function. This allows us to reduce the original problem to a system of Ham merstein integral equations. Finally, we apply the method of successive approximations to demonstrate the solvability of the nonlinear system

Keywords: Nonlinearity, uniqueness, Friedrichs inequality, existence, system of Hammerstein equations.

Introduction

The pursuit of mathematical rigor in understanding natural phenomena has prompted researchers to delve into the realm of mathematical modeling. A prominent ex ample lies in the application of linear and nonlinear odd-order equations to model diverse wave propagation processes. These include large-amplitude waves, tsunamis traversing vast oceans, nonlinear wave propagation in shallow waters, hydromagnetic wave propa gation through cold plasma, and acoustic waves within anharmonic crystals[2-7].

The following equation refers to poorly studied odd-order equations:

$$L(u) \equiv \frac{\partial^{2n+1}u}{\partial x^{2n+1}} + (-1)^n \frac{\partial u}{\partial y} = f(x, y, u(x, y), \frac{\partial u}{\partial x}, \dots, \frac{\partial^{2n}u}{\partial x^{2n}}), (MC)$$

which is called an equation with multiple characteristics (*MC*) [1].

Partial cases of the equation with MC are found in various problems of physics and mechanics; they are of great theoretical and applied importance.

This equation for *n =1* contains the well-known Korteweg-de Vries equation *(KdV*)

$$u_y + uu_x + \beta u_{xxx} = 0(KdV)$$

which is the object of research by many authors and occupies an important place in the study of nonlinear wave propagation in weakly dispersive media [2-6].

The Korteweg–de Vries (*KdV*) equation describes the evolution of weakly nonlinear long-wavelength excitations in a medium with dispersion in the high-frequency domain. The *KdV* equation is used in the study of many physical systems, such as the propagation of gravitational waves in shallow water, ion-acoustic waves in plasma, nonlinear Rossby waves in a homogeneous rotating fluid, waves in electrical circuits containing nonlinear elements, etc. [2–8].

Some characteristic features of wave propagation in dispersive media can be traced already in the linear approximation [3]:

$$u_y + \beta u_{xxx} = 0. (LKdV)$$

The *(LKdV*) equation describes sufficiently long waves in media where the limit $\frac{\omega}{k}$ (phase velocity) as $k \to 0$ has a finite value (weakly dispersive waves). The *(LKdV*) equation is called the linearized Korteweg-de Vries equation [3 - 4].

In connection with these and other practical applications, the study of boundary value problems for odd-order equations with multiple characteristics is relevant. Note that some linear boundary value problems for a linear equation with multiple charac teristics of a third-order were considered in [9-12]. A linear boundary value problem associated with a nonlinear third-order differential equation possessing multiple char acteristics was analyzed in [11]. A nonlinear boundary value problem for a third-order linear equation with multiple characteristics was studied in [13]. In [14], a nonlinear boundary value problem for a third-order nonlinear equation with multiple characteris tics was studied using the contraction mapping principle method. Some linear boundary value problems in a standard domain for a higher order equation with multiple char acteristics in continuous class function were studied in papers [1,15-16]. Work [15] also investigates the properties of potentials when the transition line is curved rather than straight. In [1,15], a linear boundary value problem was investigated for a nonlinear dif ferential equation of high odd order with multiple characteristics. Papers [17-18] stud ied a nonlinear boundary value problem for a high-order linear equation with multiple characteristics using the method of successive approximation. Reference [19] examines a nonlinear boundary value problem for a high-order nonlinear differential equation with multiple characteristics. The problem involves a nonlinear relationship between the sum of traces of (n+1) th to 2n th order x-derivatives and the trace of the desired function on the left boundary. The study employed the contraction mapping principle as a pri mary tool. This paper employs the method of successive approximations to investigate a nonlinear boundary value problem for an odd-order nonlinear equation with multiple characteristics in a curvilinear domain. The work involves a nonlinear relationship be tween the sum of traces of n th to 2n th order x-derivatives and the trace of the desired function on the right boundary

Statement of the problem

Problem A. It is required to determine in domain $D = \{(x,y): h_1(y) < x < h_2(y), 0 < y \le 1\}$ function u(x,y) that has the following properties:

1) $u(x,y) \in C_{x,y}^{2n+1,1}(D) \cap C_{x,y}^{2n,0}(\overline{D})$;that has the following properties

2) which is a regular solution to the following equation:

$$L(u) \equiv \frac{\partial^{2n+1} u}{\partial x^{2n+1}} + (-1)^n \frac{\partial u}{\partial y} = f(x,y,u(x,y)) \tag{1}$$

in domain D;

3) satisfying the following conditions

$$u(x,0) = u_0(x), h_1(0) \le x \le h_2(0), \tag{2}$$

$$\sum_{j=0}^{n} a_j \frac{\partial^{2n-j} u}{\partial x^{2n-j}}\Big|_{x=h_2(y)} = g(u(h_2(y),y),y), 0 \le y \le 1, \tag{3}$$

$$\frac{\partial^i u}{\partial x^i}\Big|_{x=h_1(y)} = \phi_i(y), 0 \le y \le 1, i = \overline{0,n}, \tag{4}$$

$$\frac{\partial^j u}{\partial x^j}\Big|_{x=h_2(y)} = \psi_j(y), 0 \le y \le 1, j = \overline{1,n-1}, \tag{5}$$

and matching conditions

$$g(u(h_2(0),0),0) = \sum_{j=0}^{n-1} a_j \frac{\partial^{2n-j} u_0(h_2(0))}{\partial x^{2n-j}},\ \varphi_i(0) = \frac{\partial^i u_0(h_1(0))}{\partial x^i},\ i = \overline{0,\ n},\ \psi_j(0) = \frac{\partial^j u_0(h_2(0))}{\partial x^j}, j = \overline{1,n-1}.$$

Uniqueness of the solution

Theorem (Uniqueness of solution). Let along with the conditions of the lemma [see (19)], the following conditions be satisfied.

Let $h_r(y) \in C^1[0;1], r = 1,2$ and for $a_n = const \ne 0, a_n = (-1)^n \alpha^{2n}, -\frac{1}{n} \le \alpha < 0, \alpha = const,$

$$(-1)^{n+1}\alpha^{2n} - h_2{}'(y) > 0, \tag{8}$$

$g(u,y), f\big(x,y,u(x,y)\big)$ be continuous functions of their arguments $0 \le y \le 1$ for any $|u| < \infty$,satisfying the following condition

$$|g(u_1,y) - g(u_2,y)| \le l_1(y)|u_1 - u_2|, \tag{9}$$

$$|f(x,y,u_1) - f(x,y,u_2)| \le l_2(x,y)|u_1 - u_2|, \tag{10}$$

$$0 < l_1(y) \le \frac{(-1)^{n+1}\alpha^{2n} - h_1'(y)}{2}, \tag{11}$$

$$0 < l_2(x,y) \le \frac{1}{2}. \tag{12}$$

Then the solution to Problem A is unique.

Proof. The uniqueness of the solution to the problem is proven by the method of energy integrals, using some elementary inequalities.

Let there be two solutions to the considered problem, u_1 and u_2. Consider their difference $w = u_1 - u_2$. With regard to w, we obtain the following problem:

$$\mathrm{L}(w) \equiv \frac{\partial^{2\tau+1} w}{\partial x^{2n+1}} + (-1)^n \frac{\partial w}{\partial y} = f(x,y,u_1(x,y)) - f(x,y,u_2(x,y)) \tag{1_0}$$

$$w(x,0) = 0, h_1(0) \le x \le h_2(0), \tag{2_0}$$

$$\sum_{j=0}^{n} a_j \frac{\partial^{2n-j} w}{\partial x^{2n-j}}\Big|_{x=h_2(y)} = g(u_1(h_2(y),y),y) - g(u_2(h_2(y),y),y), 0 \le y \le 1, \tag{3_0}$$

$$\frac{\partial^i w}{\partial x^i}\Big|_{x=h_1(y)} = 0, 0 \le y \le 1, i = \overline{0,n}, \tag{4_0}$$

$$\frac{\partial^j w}{\partial x^j}\Big|_{x=h_2(y)} = 0, 0 \le y \le 1, j = \overline{1,n-1}. \tag{5_0}$$

Let us prove that $w(x,y) \equiv 0$.

Having integrated the following identity

$$v\, w\, \mathrm{L}(w) vwL(w) \equiv vw\Big(\frac{\partial^{2n+1} w}{\partial x^{2n+1}} + (-1)^n \frac{\partial w}{\partial y}\Big) = vw(f(x,y,u_1(x,y)) - f(x,y,u_2(x,y))) \tag{13}$$

over domain D, we obtain:

$$\int_\Gamma \Bigg\{ \sum_{j=0}^{n-1} (-1)^j \frac{\partial^{2n-j} w}{\partial x^{2n-j}} \sum_{i=0}^{j} C_j^i \frac{\partial^i w}{\partial x^i} \frac{\partial^{j-i} v}{\partial x^{j-i}} + \sum_{j=0}^{n-1} (-1)^{n+j} \frac{\partial^{n-j} w}{\partial x^{n-j}} \Bigg[\sum_{i=0}^{n-1-j} C_{n+j}^i \frac{\partial^i w}{\partial x^i} \frac{\partial^{n+j-i} v}{\partial x^{n+j-i}} + \frac{1}{2} C_{n+j}^{n-j} \frac{\partial^{n-j} w}{\partial x^{n-j}} \frac{\partial^{2j} v}{\partial x^{2j}} \Bigg] + \frac{1}{2} w^2 \frac{\partial^{2n} v}{\partial x^{2n}} \Bigg\} dy -$$

$$-\frac{(-1)^n}{2}(w^2v)dx+\iint_D\sum_{j=0}^{n-1}(-1)^{n+j+1}\left(C_{n+j}^{n-1-j}+\frac{1}{2}C_{n+j}^{n-j}\right)\left(\frac{\partial^{n-j}w}{\partial x^{n-j}}\right)^2\frac{\partial^{2j+1}v}{\partial x^{2j+1}}dxdy$$

$$-\frac{1}{2}\iint_D\left(\frac{\partial^{2n+1}v}{\partial x^{2n+1}}+(-1)^n\frac{\partial v}{\partial y}\right)w^2dxdy=$$

$$=\iint_D\big(f(x,y,u_1)-f(x,y,u_2)\big)\,wvdxdy,$$

where $\Gamma=\partial D, C_n^m=\frac{n!}{m!(n-m)!}$.

Taking into account boundary conditions (2_0) - (5_0), we have

$$(-1)^{n+1}\int_0^1\sum_{j=0}^{n-1}(-\alpha)^j\frac{\partial^{2n-j}w}{\partial x^{2n-j}}wv\bigg|_{x=h_2(y)}dy+\frac{1}{2}\int_0^1\left(\frac{\partial^n w}{\partial x^n}\right)^2v\bigg|_{x=h_2(y)}dy$$

$$+\frac{1}{2}\int_0^1[h_2'(y)-(-1)^n\alpha^{2n}]w^2v|_{x=h_2(y)}dy+$$

$$+\frac{1}{2}\int_{h_1(1)}^{h_2(1)}w^2v|_{y=1}dx+\iint_D\sum_{j=0}^{n-1}(-1)^{j+1}\alpha^{2j+1}\left(C_{n+j}^{n-1-j}+\frac{1}{2}C_{n+j}^{n-j}\right)\left(\frac{\partial^{n-j}w}{\partial x^{n-j}}\right)^2vdxdy-$$

$$-\frac{1}{2}\iint_D((-1)^n\alpha^{2n+1}+\beta)\,w^2vdxdy=(-1)^n\iint_D\big(f(x,y,u_1)-f(x,y,u_2)\big)\,wvdxdy. \quad (14)$$

where $a_j=(-\alpha)^j, 0<-\alpha\le\frac{1}{n}$.

We introduce the following notation:

$$S=\frac{1}{2}\int_0^1\left(\frac{\partial^n w}{\partial x^n}\right)^2v\bigg|_{x=h_2(y)}dy+\frac{1}{2}\int_{h_1(1)}^{h_2(1)}w^2v|_{y=1}dx+$$

$$+\frac{2n+1}{2}(-\alpha)^{2n-1}\iint_D w_x{}^2v\,dxdy+\iint_D\sum_{j=0}^{n-2}(-1)^j(-\alpha)^{2j+1}\left(C_{n+j}^{n-1-j}+\frac{1}{2}C_{n+j}^{n-j}\right)\left(\frac{\partial^{n-j}w}{\partial x^{n-j}}\right)^2vdxdy. \quad (15)$$

When the condition of the lemma is satisfied, $S\ge 0$.

According to notation (15) from (14), we have

$$S=-\int_0^1\sum_{j=0}^{n-1}(-\alpha)^j\frac{\partial^{2n-j}w}{\partial x^{2n-j}}wv\bigg|_{x=h_2(y)}dy-\frac{1}{2}\int_0^1[(-1)^{n+1}\alpha^{2n}-h_2'(y)]w^2v|_{x=h_2(y)}dy+$$

$$+\frac{1}{2}\iint_D w^2vdxdy-\iint_D\big(f(x,y,u_1)-f(x,y,u_2)\big)\,wvdxdy.$$

With conditions (3_0), (9), and (10), we have

$$S\le\int_0^1\left\{l_1(y)-\frac{(-1)^{n+1}\alpha^{2n}-h_2'(y)}{2}\right\}w^2v\bigg|_{x=h_2(y)}dy+\iint_D\left\{l_2(x,y)-\frac{1}{2}\right\}w^2vdxdy.$$

When conditions (12), (13) are satisfied, we arrive at the following inequality $S\le 0$. Hence, $S=0$.

Then from (15) we obtain the following conditions:

$$\begin{cases}\frac{д^n w}{дx^n}\bigg|_{x=h_2(y)}=0{,}0\le y\le 1,\\ w(x,1)=0, h_1(1)\le x\le h_2(1),\\ w_x(x,y)=0,(x,y)\in D,\\ \frac{\partial^{n-j}w}{\partial x^{n-j}}=0,(x,y)\in D, j=\overline{0,n-2}.\end{cases}$$

Hence, we have $w(x,y)=p(y),(x,y)\in D$.

Since $w(h_1(y),y)=0{,}0\le y\le 1$, then $p(y)\equiv 0$.

Due to continuity $w(x,y)$ in $\overline{D}$we have $w(x,y)=0$.

Existence of the solution

Before proceeding to the proof of the existence of a solution to Problem A, it is necessary to study the following auxiliary problem.

Problem B. It is required to determine in domain D a regular solution

$$u(x,y)\in C_{x,y}^{2n+1,1}(D)\cap C_{x,y}^{2n,0}(\overline{D})$$

to equation

$$L(u)\equiv\frac{\partial^{2n+1}u}{\partial x^{2n+1}}+(-1)^n\frac{\partial u}{\partial y}=f(x,y) \quad (1_1)$$

satisfying the following conditions

$$u(x,\rightleftarrows\rightleftarrows 0)=u_0(x), h_1(0)\le \mathrm{x}\le h_2(0), \quad (2)$$

$$\frac{\partial^{2n}u}{\partial x^{2n}}\bigg|_{x=h_2(y)}=\phi_{2n}(y), 0\le y\le 1, \quad (3_1)$$

$$\left.\frac{\partial^i u}{\partial x^i}\right|_{x=h_1(y)} = \phi_i(y), 0 \le y \le 1, i = \overline{0,n}, \tag{4}$$

$$\left.\frac{\partial^j u}{\partial x^j}\right|_{x=h_2(y)} = \psi_j(y), 0 \le y \le 1, j = \overline{1,n-1}. \tag{5}$$

Let us construct the Green function for problem B.

The following identity holds:

$$zL(q) - qM(z) = \frac{\partial}{\partial \xi}\left\{\sum_{k-0}^{2n}(-1)^k \frac{\partial^k z}{\partial \xi^k}\frac{\partial^{2n-k} q}{\partial \xi^{2n-k}}\right\} + (-1)^n \frac{\partial}{\partial \eta}(zq), \tag{17}$$

where $M \equiv -\frac{\partial^{2n+1}}{\partial \xi^{2n+1}} - (-1)^n \frac{\partial}{\partial \eta}$ is the operator conjugate to operator $L \equiv \frac{\partial^{2n+1}}{\partial \xi^{2n+1}} + (-1)^n \frac{\partial}{\partial \eta}$,

and $z(\xi,\eta)$ and $q(\xi,\eta)$ are rather smooth functions.

Integrating identity (17) over domain D, we obtain

$$\iint_D \big(zL(q) - qM(z)\big)\, d\xi d\eta = \int_\Gamma \left(\sum_{k-0}^{2n}(-1)^k \frac{\partial^k z}{\partial \xi^k}\frac{\partial^{2n-k} q}{\partial \xi^{2n-k}}\right) d\eta - (-1)^n (zq) d\xi, \tag{18}$$

where $\Gamma = \partial D$.

Now, in formula (18), for functions q and z, we take functions $u\ (x,y)$ - any regular solution to equation $Lu = f(x,y)$ and $U(x,y;\xi,\eta)$ - fundamental solution to equation $Lu = 0$ ([1], [5], [6]).

Let domain D^ε be defined by the following inequality:

$$D^\varepsilon = \{(x,y): h_1(\eta) \le \xi \le h_2(\eta), 0 < \eta \le y - \varepsilon\}.$$

where $\varepsilon > 0$ is a sufficiently small number. Then identity (18) with regard to D^ε takes the following form:

$$\iint_{D_\varepsilon} U(x,y;\xi,\eta) f(\xi,\eta)\, d\xi d\eta = \int_0^{y-\varepsilon} \sum_{k-0}^{2n} (-1)^k \frac{\partial^k U}{\partial \xi^k}\frac{\partial^{2n-k} u}{\partial \xi^{2n-k}}\Bigg|_{\xi=h_1(\eta)}^{\xi=h_2(\eta)} d\eta + (-1)^n \int_{h_1(\eta)}^{h_2(\eta)} (Uu)|_{\eta=0}^{\eta=y-\varepsilon}\, d\xi -$$
$$-(-1)^n \int_0^{y-\varepsilon} \big(h_2'(\eta) Uu|_{\xi=h_2(\eta)} - h_1'(\eta) Uu|_{\xi=h_1(\eta)}\big)\, d\eta.$$

Here as ε tends to zero and considering the following equality ([1], [6])

$$\lim_{\varepsilon\to 0} \int_{h_1(y-\varepsilon)}^{h_2(y-\varepsilon)} U(x,y;\xi,y-\varepsilon) u\,(\xi,y-\varepsilon) d\xi = \pi u(x,y),$$

we obtain

$$(-1)^{n+1}\pi u(x,y) = \int_0^y \sum_{k=0}^{2n} (-1)^k \frac{\partial^k U}{\partial \xi^k}\frac{\partial^{2n-k} u}{\partial \xi^{2n-k}}\Bigg|_{\xi=h_1(\eta)}^{\xi=h_2(\eta)} d\eta - (-1)^n \int_{h_1(0)}^{h_2(0)} U(x,y;\xi,0) u(\xi,0)\, d\xi -$$
$$-(-1)^n \int_0^y h_2'(\eta) Uu|_{\xi=h_2(\eta)}\, d\eta + (-1)^n \int_0^y h_1'(\eta) Uu|_{\xi=h_1(\eta)}\, d\eta - \iint_D U(x,y;\xi,\eta) f(\xi,\eta)\, d\xi d\eta. \tag{19}$$

Let now $W(x,y;\xi,\eta)$ be any regular solution to the following equation:

$$-\frac{\partial^{2n+1} v}{\partial \xi^{2n+1}} - (-1)^n \frac{\partial v}{\partial \eta} = 0, \tag{20}$$

and $u\ (x,\ y)$ is any regular solution to equation $Lu = f(x,y)$. Now, assuming in formula (18) that $z = W(x,y;\xi,\eta), q = u$, we have

$$\iint_D W(x,y;\xi,\eta) f(\xi,\eta)\, d\xi d\eta = \int_0^y \sum_{k=0}^{2n} (-1)^k \frac{\partial^k W}{\partial \xi^k}\frac{\partial^{2n-k} u}{\partial \xi^{2n-k}}\Bigg|_{\xi=h_1(\eta)}^{\xi=h_2(\eta)} d\eta + (-1)^n \int_{h_1(\eta)}^{h_2(\eta)} (Wu)|_{\eta=0}^{\eta=y}\, d\xi -$$
$$-(-1)^n \int_0^y h_2'(\eta) Wu|_{\xi=h_2(\eta)}\, d\eta + (-1)^n \int_0^y h_1'(\eta) Wu|_{\xi=h_1(\eta)}\, d\eta. \tag{21}$$

From equalities (19) and (21), we find

$$(-1)^{n+1}\pi u(x,y) = \int_0^y \sum_{k=0}^{2n} (-1)^k \frac{\partial^k (U-W)}{\partial \xi^k}\frac{\partial^{2n-k} u}{\partial \xi^{2n-k}}\Bigg|_{\xi=h_1(\eta)}^{\xi=h_2(\eta)} d\eta - (-1)^n \int_{h_1(y)}^{h_2(y)} W(x,y;\xi,y) u(\xi,y)\, d\xi -$$
$$-(-1)^n \int_{h_1(0)}^{h_2(0)} \big(U(x,y;\xi,0) - W(x,y;\xi,0)\big) u(\xi,0)\, d\xi - (-1)^n \int_0^y h_2'(\eta)(U-W)u|_{\xi=h_2(\eta)}\, d\eta +$$
$$+(-1)^n \int_0^y h_1'(\eta)(U-W)u|_{\xi=h_1(\eta)}\, d\eta - \iint_D \big(U(x,y;\xi,\eta) - W(x,y;\xi,\eta)\big) f(\xi,\eta)\, d\xi d\eta. \tag{22}$$

If regular solution $W(x,y;\xi,\eta)$ to equation (20) satisfies the following conditions:

$$\left.\begin{aligned}
&W(x,y;\xi,\eta)|_{\eta=y} = 0,\\
&\left.\frac{\partial^k W}{\partial \xi^k}\right|_{\xi=h_1(\eta)} = \left.\frac{\partial^k U}{\partial \xi^k}\right|_{\xi=h_1(\eta)}, k = \overline{0,n-1},\\
&\left.\left(\frac{\partial^{2n} W}{\partial \xi^{2n}} + (-1)^n h_2'(\eta) W\right)\right|_{\xi=h_2(\eta)} = \left.\left(\frac{\partial^{2n} U}{\partial \xi^{2n}} + (-1)^n h_2'(\eta) U\right)\right|_{\xi=h_2(\eta)},\\
&\left.\frac{\partial^k W}{\partial \xi^k}\right|_{\xi=h_2(\eta)} = \left.\frac{\partial^k U}{\partial \xi^k}\right|_{\xi=h_2(\eta)}, k = \overline{1,n},
\end{aligned}\right\} \tag{23}$$

then from formula (22) we obtain

$$u(x,y)=\frac{1}{\pi}\int_0^y\sum_{k=n+1}^{2n}(-1)^{n+k+1}\frac{\partial^k G}{\partial\xi^k}\frac{\partial^{2n-k}u}{\partial\xi^{2n-k}}\Bigg|_{\xi=h_2(\eta)}d\eta+\frac{1}{\pi}\int_0^y\sum_{k=n}^{2n}(-1)^{n+k}\frac{\partial^k G}{\partial\xi^k}\frac{\partial^{2n-k}u}{\partial\xi^{2n-k}}\Bigg|_{\xi=h_1(\eta)}d\eta+$$

$$+\frac{(-1)^{n+1}}{\pi}\int_0^y G\frac{\partial^{2n}u}{\partial\xi^{2n}}\Bigg|_{\xi=h_2(\eta)}d\eta+\frac{1}{\pi}\int_{h_1(0)}^{h_2(0)}U(x,y;\xi,0)u(\xi,0)\,d\xi-\frac{1}{\pi}\int_0^y h_1'(\eta)Gu|_{\xi=h_1(\eta)}\,d\eta+$$

$$+\frac{(-1)^n}{\pi}\iint_D G(x,y;\xi,\eta)f(\xi,\eta)\,d\xi d\eta,\ (24)$$

where $G(x,y;\xi,\eta)=U(x,y;\xi,\eta)-W(x,y;\xi,\eta)$.

Formula (24) gives a solution to problem B. However, for this, we still need to prove the existence of function $W(x,y;\xi,\eta)$ that satisfies equation (20) and conditions (23).

Function $G(x,y;\xi,\eta)$ we call the Green function of problem B.

Note that for function $G(x,y;\xi,\eta)$ the same estimates as for function $U(x,y;\xi,\eta)$ are true (see [1], [6]).

$$\left.\begin{array}{l}
U(x.y;\xi,\eta)=\begin{cases}(y-\eta)^{-\frac{1}{2n+1}}f\left(\frac{x-\xi}{(y-\eta)^{\frac{1}{2n+1}}}\right),x\neq\xi,y>\eta,\\ 0,y\leq\eta.\end{cases}\\
V_m(x.y;\xi,\eta)=\begin{cases}(y-\eta)^{-\frac{1}{2n+1}}\phi_m\left(\frac{x-\xi}{(y-\eta)^{\frac{1}{2n+1}}}\right),x>\xi,y>\eta,\\ 0,y\leq\eta,\end{cases}\\
W_m(x.y;\xi,\eta)=\begin{cases}(y-\eta)^{-\frac{1}{2n+1}}\psi_m\left(\frac{x-\xi}{(y-\eta)^{\frac{1}{2n+1}}}\right),x<\xi,y>\eta,\\ 0,y\leq\eta,\end{cases}
\end{array}\right\}\quad(25)$$

where $m=\overline{1,n}$.

Let's define the following functions:

$$f(t)=\int_0^{+\infty}cos(\lambda^{2n+1}-\lambda t)\,d\lambda,-\infty<t<+\infty,$$

$$\phi_m(t)=\left\{\left[\overline{\Phi_k(t)}\right]\cap\left[\overline{\alpha_k}<0,\overline{\beta}_k>0\right]\right\}\cup\{[H_k(t)]\cap[\alpha_k<0,\beta_k>0]\}\cup\overline{\phi}(t),t>0,$$

$$\psi_m(t)=\{[\Phi_k(t)]\cap[\alpha_k>0,\beta_k>0]\}\cup\left\{\left[\overline{H_k(t)}\right]\cap\left[\overline{\alpha}_k>0,\overline{\beta}_k>0\right]\right\},t<0,$$

$$\Phi_k(t)=Im\big(\sigma_k g_k(t)\big),\overline{\Phi_k(t)}=Im\big(\overline{\sigma_k g_k}(t)\big),$$

$$H_k(t)=Re[\sigma_k g_k(t)-\overline{\sigma_0 g_0}(t)],\overline{H_k(t)}=Re\left[\overline{\sigma_k}\overline{g_k(t)}-\sigma_0 g_0(t)\right],$$

$$\overline{\phi}(t)=\begin{cases}H_n(t),ifnisanoddisnumber\\ 0,ifnisanevennumber,\end{cases},$$

$$\sigma_k=\alpha_k+i\beta_k=exp\left(\frac{n(2k+1)}{2n+1}\pi i\right),\overline{\sigma}_k=\overline{\alpha}+i\overline{\beta}_k=exp\left(\frac{2n(2k+1)+4k+1}{2(2n+1)}\pi i\right),$$

$$g_k(t)=\int_0^{+\infty}exp(\sigma_k\lambda t-\lambda^{2n+1})\,d\lambda,\overline{g_k(t)}=\int_0^{+\infty}exp(\overline{\sigma_k}\lambda t-i\lambda^{2n+1})\,d\lambda,k=\overline{0,2n},t=\frac{x-\xi}{(y-\eta)^{\frac{1}{2n+1}}}.$$

Functions $f(t),\phi_m(t),\psi_m(t)\big(m=\overline{1,n}\big)$, called Airy functions, satisfy the following equation (see [1], [5], [6]):

$$z^{(2n)}(t)-\frac{(-1)^n}{2n+1}tz(t)=0.\quad(26)$$

The following relations hold for functions $f(t),\phi_m(t),\psi_m(t)\big(m=\overline{1,n}\big)$:

As $t\to+\infty$

$$g^{(\nu)}(t)\approx\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows C_\nu^+t^{\frac{2n+1}{4n}-\frac{1}{2}}cos\left(Ct^{\frac{2n+1}{4n}}\right),\quad(27)$$

As $t\to-\infty$

$$g^{(\nu)}(t)\approx\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows\rightleftarrows C_\nu^-|t|^{\frac{2n+1}{4n}-\frac{1}{2}}exp\left(-C|t|^{\frac{2n+1}{4n}}\right),\quad(28)$$

where $\nu=0,1,2,\ldots,\mathrm{C},\mathrm{C}_\nu^-\mathrm{C}_\nu^--const>0$, and

$$\lim_{|t|\to\infty}tf(t)=0,\ \lim_{|t|\to\infty}f^{(\nu)}(t)=0,\nu=\overline{0,2n-1}.$$

The following relations hold:

$$\left.\begin{array}{l}\int_0^{+\infty}f(t)dt=\frac{n+1}{2n+1}\pi,\int_{-\infty}^0 f(t)dt=\frac{n}{2n+1}\pi,\int_{-\infty}^{+\infty}f(t)dt=\pi,\\ \int_0^{+\infty}\phi_m(t)dt=0,\int_{-\infty}^0\psi_m(t)dt=0.\end{array}\right\}\quad(29)$$

The following relations are true for functions

$U(x,y;\xi,\eta),V_m(x,y;\xi,\eta),W_m(x,y;\xi,\eta)\big(m=\overline{1,n}\big)$:

$$\left.\begin{array}{l}\left|\frac{\partial^i}{\partial x^i}(U,V_n)\right| < C_1|x-\xi|^{-\frac{2(n-i)-1}{4n}}|y-\eta|^{-\frac{2i+1}{4n}}, i=\overline{0,2n-1},\\ \left|\frac{\partial^i V_m}{\partial x^i}\right| < C_2|y-\eta|^{-\frac{i+1}{4n}} exp\left(-C\frac{|x-\xi|^{\frac{2n+1}{2n}}}{|y-\eta|^{-\frac{1}{2n}}}\right), n>1, m=\overline{1,n-1},\end{array}\right\} \quad (30)$$

as $\frac{x-\xi}{(y-\eta)^{\frac{1}{2n+1}}} \to +\infty$

$$\left|\frac{\partial^i}{\partial x^i}(U,W_m)\right| < C_3|y-\eta|^{-\frac{i+1}{2n+1}} exp\left(-C\frac{|x-\xi|^{\frac{2n+1}{2n}}}{|y-\eta|^{\frac{1}{2n}}}\right), i=\overline{0,2n-1}, m=\overline{1,n}, \quad (31)$$

as $\frac{x-\xi}{(y-\eta)^{\frac{1}{2n+1}}} \to -\infty, C, C_1, C_2, C_3 - const > 0.$

Solution to problem (20), (23) is sought in the following form:

$$W(x,y;\xi,\eta)=\int_\eta^y U(h_2(\tau),\tau;\xi,\eta)\alpha_0(x,y,\tau)d\tau +$$
$$+\int_\eta^y\sum_{m=1}^n W_m(h_2(\tau),\tau;\xi,\eta)\alpha_m(x,y,\tau)d\tau + \int_\eta^y\sum_{m=1}^n V_m(h_1(\tau),\tau;\xi,\eta)\beta_m(x,y,\tau)d\tau \ . \quad (32)$$

Obviously, function $W(x,y;\xi,\eta)$ satisfies condition $W(x,y;\xi,\eta)|_{\eta=y}=0$.

Satisfying the boundary conditions (23) we obtain

$$\left.\frac{\partial^k U(x,y;\xi,\eta)}{\partial\xi^k}\right|_{\xi=h_1(\eta)} = \int_\eta^y\frac{\partial^k U(h_2(\tau),\tau;h_1(\eta),\eta)}{\partial\xi^k}\alpha_0(x,y,\tau)d\tau + \int_\eta^y\frac{\partial^k U(h_1(\tau),\tau;h_1(\eta),\eta)}{\partial\xi^k}\beta_n(x,y,\tau)d\tau +$$
$$+\int_\eta^y\sum_{m=1}^n\frac{\partial^k V_m(h_2(\tau),\tau;h_1(\eta),\eta)}{\partial\xi^k}\alpha_m(x,y,\tau)d\tau + \int_\eta^y\sum_{m=1}^{n-1}\frac{\partial^k W_m(h_1(\tau),\tau;h_1(\eta),\eta)}{\partial\xi^k}\beta_m(x,y,\tau)d\tau\ ,\ \ k=\overline{0,n-1},$$

(34)

$$\left.\frac{\partial^k U(x,y;\xi,\eta)}{\partial\xi^k}\right|_{\xi=h_2(\eta)} = \int_\eta^y\frac{\partial^k U(h_2(\tau),\tau;h_2(\eta),\eta)}{\partial\xi^k}\alpha_0(x,y,\tau)d\tau + \int_\eta^y\frac{\partial^k U(h_1(\tau),\tau;h_2(\eta),\eta)}{\partial\xi^k}\beta_n(x,y,\tau)d\tau +$$
$$+\int_\eta^y\sum_{m=1}^n\frac{\partial^k V_m(h_2(\tau),\tau;h_2(\eta),\eta)}{\partial\xi^k}\alpha_m(x,y,\tau)d\tau + \int_\eta^y\sum_{m=1}^{n-1}\frac{\partial^k W_m(h_1(\tau),\tau;h_2(\eta),\eta)}{\partial\xi^k}\beta_m(x,y,\tau)d\tau\ ,\ \ k=\overline{1,n},$$

(34)

$$\left.\left(\frac{\partial^{2n}U}{\partial\xi^{2n}}+(-1)^n h_2'(\eta)U\right)\right|_{\xi=h_2(\eta)} = \int_\eta^y\frac{\partial^{2n}U(h_2(\tau),\tau;h_2(\eta),\eta)}{\partial\xi^{2n}}\alpha_0(x,y,\tau)d\tau + \int_\eta^y\frac{\partial^{2n}U(h_1(\tau),\tau;h_2(\eta),\eta)}{\partial\xi^{2n}}\beta_n(x,y,\tau)d\tau +$$
$$+\int_\eta^y\sum_{m=1}^n\frac{\partial^{2n}V_m(h_2(\tau),\tau;h_2(\eta),\eta)}{\partial\xi^{2n}}\alpha_m(x,y,\tau)d\tau + \int_\eta^y\sum_{m=1}^{n-1}\frac{\partial^{2n}W_m(h_1(\tau),\tau;h_2(\eta),\eta)}{\partial\xi^{2n}}\beta_m(x,y,\tau)d\tau +$$
$$+(-1)^n h_2'(\eta)\left\{\int_\eta^y U(h_2(\tau),\tau;h_2(\eta),\eta)\alpha_0(x,y,\tau)d\tau+\int_\eta^y U(h_1(\tau),\tau;h_2(\eta),\eta)\beta_n(x,y,\tau)d\tau\right\}+$$
$$+(-1)^n h_1'(\eta)\left\{\int_\eta^y\sum_{m=1}^n V_m(h_2(\tau),\tau;h_2(\eta),\eta)\alpha_m(x,y,\tau)d\tau + \int_\eta^y\sum_{m=1}^{n-1}W_m(h_1(\tau),\tau;h_2(\eta),\eta)\beta_m(x,y,\tau)d\tau\right\},$$

(35)

This is a system of Volterra integral equations for unknown functions $\alpha_k(x,y,\tau),\ \beta_m(x,y,\tau)\ \left(k=\overline{0,n},\ m=\overline{1,n}\right)$.

If $\alpha_k(x,y,\tau)\in C^1(D),\ \beta_m(x,y,\tau)\in C(D),\ m=\overline{1,n},\ k=\overline{0,n}$, then the system (33)-(35) is solvable.

We now proceed to problem A.

Theorem. Let, along with the conditions of the uniqueness theorem, the following conditions be satisfied:

$$u_0(x)\in C^{2n+1}[h_1(0;h_2(0))],\ \varphi_i(y)\in C^1[0,1],\ i=\overline{0,n},\ \ \psi_j(y)\in C[0,1],\ \ j=\overline{1,n-1},$$
$$\frac{\partial}{\partial y}(f(x,y,u))\in C(\overline{D}), f(x,0,u(x,0))=0,$$

and for $y \in [0;1]$for any $|\tau(y)| < \infty$ the following inequalities hold:

$$|g(\tau(y),y)| < N_1, |g_y(\tau(y),y)| < N_2, |g_\tau(\tau(y),y)| < N_3,$$

and $f(x,y,u) \in C(D)$ for $(x,y) \in D$ and for $|u| < \infty$

$$|f(x,y,u)| < M, |f_x(x,y,u)| < M_1, |f_y(x,y,u)| < M_2, |f_u(x,y,u)| < M_3.$$

Then the solution to problem (1) - (5) exists.

Proof. From (24), we find

$$u(x,y) = \frac{(-1)^{n+1}}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)\, g(u(h_2(\eta),\eta),\eta)d\eta$$
$$+\frac{(-1)^{n+1}}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)\sum_{j=1}^n a_j \frac{\partial^{2n-j}u}{\partial x^{2n-j}}\bigg|_{\xi=h_2(\eta)} d\eta +$$
$$+\frac{(-1)^n}{\pi}\iint_D G(x,y;\xi,\eta)f\big(\xi,\eta,u(\xi,\eta)\big)\,d\xi d\eta + F(x,y), \tag{36}$$

where

$$F(x,y) = \frac{1}{\pi}\int_0^y \sum_{k=n}^{2n}(-1)^{n+k}\frac{\partial^k G}{\partial \xi^k}\bigg|_{\xi=h_1(\eta)} \phi_{2n-k}(\eta)d\eta - \frac{1}{\pi}\int_0^y \sum_{k=n+1}^{2n-1}(-1)^{n+k+1}\frac{\partial^k G}{\partial \xi^k}\bigg|_{\xi=h_2(\eta)} \psi_{2n-k}(\eta)\,d\eta +$$
$$+\frac{1}{\pi}\int_{h_1(0)}^{h_2(0)} G(x,y;\xi,0)u_0(\xi)\,d\xi + \frac{1}{\pi}\int_0^y h_2'(\eta)G(x,y;h_2(\eta),\eta)\psi_0(\eta)\,d\eta. \tag{37}$$

The solution to problem A is sought in the form (36).

Let us introduce the following notation:

$$u(h_2(y),y) = \tau(y), \sum_{j=1}^n a_j \frac{\partial^{2n-j}u}{\partial x^{2n-j}}\bigg|_{x=h_2(y)} = \nu(y). \tag{38}$$

Passing to the limit for $x \to h_2(y)$ in (36), according to notation (38), we obtain:

$$\tau(y) = \frac{(-1)^{n+1}}{\pi}\int_0^y G(h_2(y),y;h_2(\eta),\eta)\, g(\tau(\eta),\eta)d\eta + \frac{(-1)^n}{\pi}\int_0^y G(h_2(y),y;h_2(\eta),\eta)\,\nu(\eta)d\eta +$$
$$+\frac{(-1)^n}{\pi}\iint_D G(h_2(y),y;\xi,\eta)f\big(\xi,\eta,u(\xi,\eta)\big)\,d\xi d\eta + F(h_2(y),y). \tag{39}$$

Differentiating (36) with respect to *x* 2*n*-*j* times $\big(j = \overline{1,n}\big)$ and multiplying by a_j,

then adding, and passing to the limit for $x \to h_2(y)$, we obtain:

$$\nu(y) = \frac{(-1)^{n+1}}{\pi}\int_0^y \sum_{j=1}^n a_j \frac{\partial^{2n-j}G(h_2(y),y;h_2(\eta),\eta)}{\partial x^{2n-j}} g(\tau(\eta),\eta)d\eta$$
$$+\frac{(-1)^n}{\pi}\int_0^y \sum_{j=1}^{n-1} a_j \frac{\partial^{2n-j}G(h_2(y),y;h_2(\eta),\eta)}{\partial x^{2n-j}}\nu(\eta)d\eta +$$
$$+\frac{(-1)^n}{\pi}\iint_D \sum_{j=1}^{n-1} a_j \frac{\partial^{2n-j}G(h_2(y),y;\xi,\eta)}{\partial x^{2n-j}} f\big(\xi,\eta,u(\xi,\eta)\big)\,d\xi d\eta + \sum_{j=1}^{n-1} a_j \frac{\partial^{2n-j}F(h_2(y),y)}{\partial x^{2n-j}}, \tag{40}$$

System (36), (39) - (40) It is a system of nonlinear Hammerstein integral equations with respect to $u(x,y), \tau(y)$ and $\nu(y)$.

We will prove the unique solvability of this system using the method of successive approximations.

Let $g(\tau(y),y)$ and $f(x,y,u(x,y))$ as known funktion, then from (40) we will find funtion $\nu(y)$.

$$\nu(y) = \frac{(-1)^n}{\pi}\int_0^y \sum_{j=1}^{n-1} a_j \frac{\partial^{2n-j}G(h_2(y),y;h_2(\eta),\eta)}{\partial x^{2n-j}}\nu(\eta)d\eta + F_1(h_2(y),y), \tag{41}$$

where

$$\left|\frac{\partial^j G(h_2(y),y;h_2(\eta),\eta)}{\partial x^j}\right| < C|y-\eta|^{-\frac{2(n-j)-1}{2(2n+1)}}, j = \overline{n+1,2n-1} \tag{42}$$

It is easy to show that for continuous functions $F_1(h_2(y),y)$ there is a unique solution in the class of continuous functions that admit a representation of the form

$$\nu(y) = \int_0^y \sum_{j=n}^{2n-1} R(y,\eta)F_{1j}(\eta)d\eta + F_1(y), \tag{43}$$

where the kernel resolvent - $R(y,\eta)$ has the same estimate as (42).

According to (43) out of (36) we have

$$u(x,y) = \frac{(-1)^{n+1}}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)\sum_{j=1}^n a_j\left[\int_0^\eta \sum_{j=n}^{2n-1} R(y,\sigma)\,F_{1j}(\sigma)d\sigma + F_1(\eta)\right]d\eta +$$
$$+\frac{(-1)^n}{\pi}\iint_D G(x,y;\xi,\eta)f\big(\xi,\eta,u(\xi,\eta)\big)\,d\xi d\eta + F(x,y). \tag{44}$$

Passing to the limit for $x \to h_2(y)$ in (44), according to notation (38), we obtain:

$$\tau(y) = \int_0^y K(y,\eta)\, g(\tau(\eta),\eta)d\eta + E(y), \tag{45}$$

where

$$K(y,\eta)=\frac{(-1)^{n+1}}{\pi}\int_0^y G(h_2(y),y;h_2(\eta),\eta)+$$

$$+\frac{(-1)^n}{\pi^2}\int_0^\eta\sum_{j=n}^{2n-1}G(h_2(y),y;h_2(\sigma),\sigma)\,a_j(\sigma)\frac{\partial^k G(h_2(\sigma),\sigma;h_2(\eta),\eta)}{\partial x^k}d\sigma+$$

$$+\frac{1}{\pi}\int_0^\eta\sum_{j=n}^{2n-1}G(h_2(y),y;h_2(\zeta),\zeta)\,a_j(\zeta)\left[\int_0^\eta\sum_{k=n}^{2n-1}R(\zeta,\sigma)\frac{\partial^k G(h_2(\sigma),\omega;h_2(\eta),\eta)}{\partial x}d\sigma\right]d\zeta$$

It is nonlinear Hammerstein integral equation with $\tau(y)$.

Equation (45) can be solved by the method of successive approximations.

Let's now investigate the equation (36).

$$u(x,y)=P(x,y)+\frac{(-1)^n}{\pi}\iint_D G(x,y;\xi,\eta)f\big(\xi,\eta,u(\xi,\eta)\big)\,d\xi d\eta, \quad (46)$$

where

$$P(x,y)=\frac{(-1)^{n+1}}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)\,g(\tau(\eta),\eta)d\eta+\frac{(-1)^{n+1}}{\pi}\int_0^y G(x,y;h_2(\eta),\eta)\,\nu(\eta)d\eta+F(x,y).$$

Equation (46) also can be solved iteratively using the method of successive approximations.

Let

$$|P(x,y)|\le N,|f(x,y,u)|\le M,|u|\le N+1.$$

Assuming that

$$u^{(0)}(x,y)=P(x,y).$$

$$u^{(k)}(x,y)=P(x,y)+\frac{(-1)^n}{\pi}\iint_D G(x,y;\xi,\eta)f\left(\xi,\eta,u^{(k-1)}(\xi,\eta)\right)d\xi d\eta, \quad (47)$$

Assuming n = 1 in equation (47) and using the estimate

$$|G(x,y;\xi,\eta)|<C_1|y-\eta|^{-\frac{1}{4n}}|x-\xi|^{-\frac{2n-1}{4n}}$$

we have

$$\left|u^{(1)}\right|<N+CMy^{\frac{4n-1}{4n}}.$$

In order for $(x,y)\in\overline{D}$ the value of the argument u of the function f(*x,y,u*) to satisfy inequality (47), under which f is limited, it is necessary to fulfill the conditions

$$N+CMy^{\frac{4n-1}{4n}}\le N+1$$

or

$$y\le\left(\frac{1}{CM}\right)^{\frac{4n}{4n-1}} \quad (48)$$

For n=2 from (47) we find

$$\left|u^{(2)}\right|<N+CMy^{\frac{4n-1}{4n}}.$$

According to (48)

$$\left|u^{(2)}\right|<N+1.$$

that is, when repeating (in the second) approximation of the argument *u*, the function *f* does not leave the limited region (46).

Applying the complete method of induction, we conclude that none of the successive approximations will leave the region (46) if condition (48) are met.

Now, we will show that the limit of the sequence $\{u^{(n)}\}$ exist.

To establish this, it suffices to prove the convergence of the series

$$u^{(0)}+\left(u^{(1)}-u^{(0)}\right)+\left(u^{(2)}-u^{(1)}\right)+\ldots+\left(u^{(k)}-u^{(k-1)}\right)+\ldots \quad (49)$$

Let's estimate the absolute values of the terms in series (49).

$$\left.\begin{array}{l}
\left|u^{(0)}\right|<N,\\
\left|u^{(1)}-u^{(0)}\right|\le\frac{CM}{\pi}B\left(1,\frac{4n-1}{4n}\right)y^{\frac{4n-1}{4n}},\\
\left|u^{(2)}-u^{(1)}\right|\le\frac{C^2lM}{\pi^2}B\left(2-\frac{1}{4n},\frac{4n-1}{4n}\right)B\left(1,\frac{4n-1}{4n}\right)y^{2\left(1-\frac{1}{4n}\right)},\\
\left|u^{(3)}-u^{(2)}\right|\le\frac{C^3l^2M}{\pi^3}B\left(3-\frac{2}{4n},\frac{4n-1}{4n}\right)B\left(2-\frac{1}{4n},\frac{4n-1}{4n}\right)B\left(1,\frac{4n-1}{4n}\right)y^{3\left(1-\frac{1}{4n}\right)},\\
\ldots.\\
\left|u^{(k)}-u^{(k-1)}\right|\le\frac{C^kl^{k-1}M}{\pi^k}\prod_{q=1}^{k}B\left(q-\frac{q-1}{4n},\frac{4n-1}{4n}\right)y^{k\left(1-\frac{1}{4n}\right)},\\
\ldots
\end{array}\right\} \quad (50)$$

where

$$B(a,b)=\int_0^1 x^{a-1}(1-x)^{b-1}dx$$

Beta function, P-multiplication sign.

It is evident that the absolute value of each term in series (50) is no greater than the corresponding term in the power series.

$$\sum_{k=1}^{\infty}\frac{c^k l^{k-1} M}{\pi_k}\prod_{q=1}^{k} B\left(q-\frac{q-1}{4n},\frac{4n-1}{4n}\right) y^{k\left(1-\frac{1}{4n}\right)}. \tag{51}$$

We will establish the convergence of the series.

Applying the D'Alembert criterion, it follows that

$$\lim_{k\to\infty}\left|\frac{u_{k+1}}{u_k}\right| = \lim_{k\to\infty}\frac{Cl}{\pi}B\left(k-\frac{1}{4n},1-\frac{1}{4n}\right)y^{\frac{4n-1}{4n}} = 0.$$

Since the series (51) converges uniformly, it follows that it also converges absolutely and uniformly.

Taking the limit under the integral sign in equation (49), we obtain

$$u(x,y) = P(x,y) + \frac{(-1)^n}{\pi}\iint_D G(x,y;\xi,\eta) f\big(\xi,\eta,u(\xi,\eta)\big)\, d\xi d\eta.$$

We have shown that a solution to the problem exists within the domain $D_0 = \{h_1(y) \le x \le h_2(y), 0 < y \le y_0\}$ even though the problem was originally formulated in the domain $D = \{(x,y): h_1(y) < x < h_2(y), 0 < y \le 1\}$.The domains D0 and D are distinct.

It is evident that the problem is fully resolved if $y_0 \ge 1$.

If $y_0 <1$, then the obtained solution can be extended.This can be achieved as follows. We examine the problem within the domain $D_1 = D\backslash D_0$.

Employing the aforementioned scheme to solve the problem within the designated region, we arrive at the nonlinear integral Volterra equation

$$u(x,y) = P(x,y) + \frac{(-1)^n}{\pi}\iint_{D_1} G(x,y;\xi,\eta) f\big(\xi,\eta,u(\xi,\eta)\big)\, d\xi d\eta.$$

By employing the same argument as before, this integral equation can be solved iteratively using the method of successive approximations within the specified domain

$$D_1 = \{h_1(y) \le x \le h_2(y), y_0 < y \le y_1\},$$

where

$$y_1 \le \left(\frac{1}{CM}\right)^{\frac{4n}{4n-1}} + y_0 \text{ or } y_1 \le 2\left(\frac{1}{CM}\right)^{\frac{4n}{4n-1}}.$$

If it turns out that after this $y < 1$, then the above procedure can be repeated. Since after each step we advance by a positive value $\delta = y_{k+1} - y_k = \left(\frac{1}{CM}\right)^{\frac{4n}{4n-1}}$, then after a finite number of steps we will reach the domain D. Therefore, we conclude that the solution to problem (1)-(5) can be extended to the interval [0, 1] along the y-axis.

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MOLECULAR BIOLOGY

MYCOREMEDIATION: THE ROLE OF FUNGI IN THE REMEDIATION OF SOILS CONTAMINATED WITH HEAVY METALS AND RADIONUCLIDES

Arabova G.
Ph.D. in Biology, lecturer, Azerbaijan State Pedagogical University
Alibayli N.
Lecturer, Azerbaijan State Pedagogical University
DOI: 10.5281/zenodo.21914149

Abstract

Rapid industrialization, urbanization, mining activities, nuclear accidents, and military conflicts have led to the accumulation of heavy metals and radionuclides in terrestrial ecosystems. Soil contamination with toxic elements such as cadmium, lead, mercury, uranium, cesium, and strontium poses serious threats to biodiversity, agricultural productivity, and human health. Conventional remediation technologies are often expensive and may generate secondary pollutants. In recent decades, mycoremediation—the use of fungi for environmental cleanup—has emerged as an environmentally friendly and cost-effective alternative. Owing to their extensive mycelial networks, high metabolic activity, and remarkable capacities for biosorption, bioaccumulation, and biodegradation, fungi play a crucial role in the restoration of contaminated ecosystems.

Keywords: mycoremediation; fungi; heavy metals; radionuclides; biosorption; bioremediation.

Environmental pollution has become one of the most pressing global challenges of the twenty-first century. Anthropogenic activities, including mining, metallurgy, industrial manufacturing, fossil fuel combustion, agricultural intensification, and the widespread use of nuclear technologies, have substantially increased the concentrations of toxic contaminants in soils and aquatic ecosystems (Gadd, 2007; Singh & Ward, 2004).

Heavy metals such as cadmium (Cd), lead (Pb), mercury (Hg), chromium (Cr), nickel (Ni), copper (Cu), and arsenic (As) are persistent environmental pollutants that accumulate in ecosystems and enter the food chain, posing serious risks to ecological integrity and human health (Nagajyoti et al., 2010). Likewise, radionuclides released into the environment through nuclear weapons testing, accidents at nuclear power plants, and military activities may persist for decades because of their long half-lives, resulting in prolonged environmental contamination (IAEA, 2018).

Conventional physicochemical remediation technologies, including excavation, vitrification, and chemical extraction, are generally expensive, energy-intensive, and may adversely affect the natural structure and fertility of soils. Consequently, biological remediation strategies, particularly fungus-based technologies, have attracted considerable scientific attention as sustainable and environmentally friendly alternatives (Harms et al., 2011).

Mycoremediation is defined as the use of fungi and their enzymatic systems to remove, immobilize, transform, or detoxify environmental contaminants (Stamets, 2005; Gadd, 2007). Owing to their exceptional metabolic versatility and adaptability, fungi have emerged as highly promising biological agents for the remediation of contaminated environments.

The effectiveness of fungi in mycoremediation is largely attributed to their distinctive biological characteristics. Fungal hyphae develop extensive mycelial networks capable of penetrating soil particles and reaching contaminants that are inaccessible to many other microorganisms (Fomina & Gadd, 2014). The large surface area of the mycelium significantly enhances the adsorption, sequestration, and immobilization of toxic substances.

Furthermore, the fungal cell wall consists primarily of chitin, glucans, proteins, melanin, and other polysaccharides. Functional groups such as amino, carboxyl, phosphate, and hydroxyl groups present within the cell wall readily interact with metal ions and radionuclides, facilitating their adsorption and immobilization (Gadd, 2009).

Another key feature underlying the efficiency of mycoremediation is the production of extracellular enzymes. Numerous fungal species synthesize enzymes, including laccases, lignin peroxidases, manganese peroxidases, cellulases, and oxidases, which catalyze the transformation, detoxification, and degradation of a broad spectrum of environmental contaminants (Pointing, 2001; Singh, 2006).

The principal mechanisms involved in fungal-mediated remediation include biosorption, bioaccumulation, biotransformation, and biomineralization (Fomina & Gadd, 2014).

Biosorption is the passive binding of metal ions to the fungal cell wall. This process occurs rapidly, does not require metabolic energy, and is mediated by the functional groups present on the cell surface. Species such as *Aspergillus niger*, *Penicillium chrysogenum*, and *Rhizopus arrhizus* have demonstrated a remarkable capacity to adsorb heavy metals, including lead, cadmium, and chromium (Volesky, 2007).

Bioaccumulation involves the active transport of contaminants into fungal cells, where they are subsequently sequestered within vacuoles or complexed with metal-binding proteins such as metallothioneins. This intracellular sequestration substantially reduces the mobility and toxicity of contaminants in the surrounding environment.

Biotransformation refers to the enzymatic conversion of toxic compounds into less hazardous chemical forms through oxidation–reduction (redox) reactions and other metabolic pathways, thereby decreasing their environmental toxicity.

Biomineralization is the process by which fungi promote the precipitation of contaminants as insoluble mineral complexes. Oxalic acid secreted by numerous fungal species reacts with metal ions to form insoluble metal oxalates, effectively reducing their bioavailability, mobility, and ecological risk (Gadd, 2007).

The remediation potential of fungal species varies considerably depending on differences in cell wall composition, melanin content, enzymatic activity, physiological characteristics, and ecological adaptations. These variations determine their capacity to tolerate, immobilize, transform, and remove heavy metals and radionuclides from contaminated environments.

Fungi	**Primary contaminant**	**Dominant mechanism**	**Application area**
Aspergillus niger	Cd, Pb, Cr	Biosorption	Industrial wastewater treatment
Penicillium chrysogenum	Pb, Cu	Bioaccumulation	Soil remediation
Trichoderma harzianum	Cd, Ni	Biosorption and detoxification	Agricultural soils
Pleurotus ostreatus	Heavy metals and pesticides	Enzymatic degradation	Ecosystem restoration
Phanerochaete chrysosporium	Organic Pollutants and Radionuclides	Ligninolytic Activity	Industrial sites
Cladosporium sphaerospermum	Radionuclides	Melanin-Mediated Adsorption	Nuclear facilities
Cryptococcus neoformans	Ionizing radiation	Radiotropism	Space biotechnology

In soils contaminated with heavy metals, cadmium (Cd) and lead (Pb) are among the most hazardous pollutants to the environment (Nagajyoti et al., 2010; Gadd, 2010). Species belonging to the genera *Aspergillus*, *Penicillium*, and *Trichoderma* exhibit a high tolerance to these metals and possess considerable remediation potential (Fomina & Gadd, 2014; Gadd, 2007). Hexavalent chromium [Cr(VI)] is highly toxic and carcinogenic. Certain fungal species can reduce Cr(VI) to the less toxic trivalent form, Cr(III), thereby significantly decreasing its environmental toxicity (Acevedo-Aguilar et al., 2008). In addition, some fungi have evolved specialized resistance mechanisms that enable them to survive and function in environments contaminated with mercury (Hg) and arsenic (As), making them promising candidates for the bioremediation of these hazardous pollutants (Gadd, 2007; Baldrian, 2003).

Studies on the interactions between fungi and radionuclides expanded significantly following the Chernobyl Nuclear Power Plant accident, which highlighted the potential role of fungi in the remediation of radioactive contamination. Radiotrophic fungal species such as *Cladosporium sphaerospermum*, *Cryptococcus neoformans*, and *Wangiella dermatitidis* have been isolated from highly radioactive environments (Dadachova et al., 2007). These fungi contain high concentrations of melanin, a pigment capable of absorbing ionizing radiation and potentially participating in energy conversion processes. Fungi have demonstrated the ability to immobilize radionuclides such as uranium (U), cesium-137 (^{137}Cs), strontium-90 (^{90}Sr), and plutonium (Pu) (Gadd, 2007; Fomina & Gadd, 2014). Their adsorption efficiency depends on several factors, including environmental pH, temperature, and fungal species. Studies conducted in the Chernobyl and Fukushima exclusion zones have revealed a rich diversity of fungal communities adapted to conditions of chronic radiation exposure, highlighting their remarkable resilience and their potential for application in the bioremediation of radioactively contaminated environments (Dighton et al., 2008; Zhdanova et al., 2004).

White-rot fungi such as *Pleurotus ostreatus*, *Phanerochaete chrysosporium*, and *Trametes versicolor* produce ligninolytic enzymes capable of degrading a wide range of environmental contaminants. The nonspecific nature of these enzymes enables the transformation of both organic pollutants and metal-containing complexes. Current scientific evidence indicates that white-rot fungi, together with melanin-producing fungi, represent some of the most promising groups for the development of next-generation bioremediation technologies (Harms et al., 2011; Gadd, 2010).

The principal applications of mycoremediation include the restoration of mining areas, the rehabilitation of industrial sites, the treatment of radioactive waste, the improvement of agricultural soils, water purification, and ecosystem restoration (Singh, 2006; Fomina & Gadd, 2014). Mycorrhizal fungi, when used in association with plants, enhance plant growth, improve nutrient uptake, and increase tolerance to environmental stress in contaminated environments, thereby contributing to more effective remediation. Recent advances in molecular biology, genomics, synthetic biology, and biotechnology have created new opportunities for improving fungal-based remediation technologies. Future research is expected to focus on the comprehensive characterization of metal-tolerant fungi, the genetic engineering of fungal strains, the development of fungal consortia, the integration of nanotechnology with mycoremediation systems, and the implementation of artificial intelligence–assisted environmental monitoring to optimize remediation processes (Harms et al., 2011; Cairns et al., 2018). Despite these promising developments, several challenges continue to limit the large-scale application of mycoremediation. These include the variability of environmental conditions, competition with indigenous microorganisms, the relatively slow rate of remediation, the potential remobilization of accumulated contaminants into the environment, and the limited availability of large-scale field studies. Addressing these constraints will be essential for the successful implementation of mycoremediation as a sustainable strategy for environmental restoration.

Mycoremediation represents a promising and environmentally sustainable approach for mitigating contamination caused by heavy metals and radionuclides. Through mechanisms such as biosorption, bioaccumulation, biomineralization, and enzymatic transformation, fungi are capable of removing or immobilizing toxic substances from soil and aquatic environments (Fomina & Gadd, 2014). In particular, the remarkable adaptability of radiotrophic fungi inhabiting radioactive environments highlights their significant potential for future ecological and biotechnological applications. Future research integrating microbiology, ecology, biotechnology, and environmental engineering is expected to facilitate the development of more efficient and scalable mycoremediation technologies (Cairns et al., 2018). Such interdisciplinary efforts will contribute to improving environmental restoration strategies and promoting sustainable management of contaminated ecosystems.

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PEDAGOGY

ETHICAL PE RSPPECTIVES OF USING AI IN EDUCATION

Huseynli A.
Azerbaijan State Pedagogical University,
Foreign Languages Centre
https://orcid.org/0000-0002-4571-4678
DOI: 10.5281/zenodo.21914172

Abstract

The integration of Artificial Intelligence (AI) in education has revolutionized teaching and learning processes, offering personalized learning experiences, administrative efficiencies, and innovative pedagogical approaches. However, the deployment of AI in educational settings raises significant ethical concerns that merit careful examination. These concerns encompass issues of privacy, data security, bias, fairness, transparency, and the potential for AI to influence educational equity. While AI holds the promise of transforming education for the better, it also poses risks of exacerbating existing disparities and infringing on students' rights. This article explores the ethical perspectives surrounding the use of AI in education by analyzing various dimensions such as privacy and data protection, bias and fairness, and transparency and accountability. It emphasizes the importance of establishing ethical guidelines and policies to ensure AI's responsible use, safeguarding the rights and well-being of learners, educators, and institutions. As AI continues to evolve, fostering an ethical framework becomes crucial to harness its benefits while minimizing potential harms. This discussion aims to provide a balanced view of the ethical challenges and opportunities presented by AI in education, encouraging stakeholders to prioritize ethical considerations in AI-driven educational innovations.

Keywords: artificial intelligence, education, data, ethical, students, outcomes, privacy.

Privacy and Data Security in AI-Driven Education

One of the most pressing ethical concerns related to AI in education is the collection and use of vast amounts of data. AI systems rely heavily on data to personalize learning experiences, track student progress, and optimize educational content. However, this data often includes sensitive information such as students' personal details, academic records, behavioral patterns, and even biometric data. The ethical dilemma centers on how this data is collected, stored, and used, and whether students and their guardians are adequately informed and give genuine consent.

Data security is paramount to prevent unauthorized access, breaches, and misuse of personal information. [3] Institutions must implement robust cybersecurity measures and adhere to legal frameworks like GDPR or FERPA to protect student privacy. Moreover, there is a risk that AI algorithms may inadvertently perpetuate biases present in the training data, leading to unfair treatment or discrimination against certain student groups. For example, biased algorithms could influence grading, recommendation systems, or access to resources, thereby undermining educational equity.

Establishing clear policies on data privacy, ensuring transparency about data usage, and fostering a culture of ethical data management are essential steps toward safeguarding learners' rights. [1.6] Ethical AI implementation requires ongoing vigilance to balance the benefits of data-driven personalization with the fundamental right to privacy.

Bias, Fairness, and Equity in AI-Enabled Education

AI systems are only as good as the data they are trained on, which means they can inadvertently encode societal biases and stereotypes. In education, this can lead to unfair outcomes, such as differential treatment of students based on gender, ethnicity, socioeconomic status, or disability. [7.22] Such biases threaten the core ethical principles of fairness and equality, potentially widening the educational divide.

For instance, AI-powered testing or admissions algorithms might favor certain demographics over others if training data reflect existing societal prejudices. This raises ethical questions about the role of AI in either mitigating or reinforcing systemic inequalities. To address this, developers and educators must prioritize fairness in AI design, ensuring that algorithms are tested for bias and are regularly audited for discriminatory outcomes. [5.8]

Promoting inclusivity and equity also involves involving diverse stakeholders in AI development and decision-making processes. Ethical AI in education should aim to provide equal opportunities for all learners, regardless of their background. Transparency in how AI systems make decisions and accountability for outcomes are crucial for fostering trust and fairness.

Transparency, Accountability, and the Role of Stakeholders

The opacity of many AI algorithms presents another significant ethical challenge. Many AI models, especially deep learning systems, operate as "black boxes," making it difficult to understand how they arrive at specific decisions. In educational contexts, this lack of transparency can undermine trust and raise questions about accountability.

Stakeholders—including students, parents, educators, administrators, and policymakers—must have clarity about how AI tools function, how decisions are made, and what implications they carry.[4.154] Establishing clear guidelines for explainability and interpretability of AI systems is vital to ensure that users can scrutinize and challenge automated decisions when necessary.

Accountability mechanisms should also be put in place to address errors or adverse outcomes resulting from AI use. This involves defining responsibility among developers, institutions, and policymakers for ensuring ethical AI deployment. Continuous monitoring, evaluation, and stakeholder engagement are necessary components of responsible AI integration. Ultimately, fostering transparency and accountability in AI-driven education promotes trust, empowers users, and aligns technological advancement with ethical principles.

Conclusion: While the integration of AI into education offers remarkable opportunities to enhance learning experiences and operational efficiencies, it simultaneously presents significant ethical challenges that must not be overlooked. Addressing concerns related to privacy, bias, transparency, and equity is essential to ens ure that AI serves as a tool for inclusive and fair education rather than exacerbating existing disparities. Developing comprehensive ethical guidelines and policies is crucial for safeguarding the rights and well-being of all educational stakeholders. As AI technology advances, a proactive focus on ethical considerations will be instrumental in maximizing its benefits while mitigating potential harms. Ultimately, fostering an ethical framework for AI in education will enable stakeholders to harness its transformative potential responsibly, promoting an equitable and trustworthy educational future.

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PHILOLOGY

WOMEN'S ISSUES IN NADEEM ASLAM'S MAPS FOR LOST LOVERS

Azizaliyeva B.
Dr.
Azerbaijan National Academy of Sciences, Institute of Literature named after Nizami Ganjavi
ORCİD: 0000-0001-5353-4856
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Abstract

The novel portrays the lives and experiences of Pakistani Muslims in Britain and explores how they construct and negotiate their identities within diverse cultural contexts. The female characters represented in the novel are shaped through multiple perspectives in accordance with Nadeem Aslam's artistic vision, including the themes of free love and individual choice, domestic violence, gender inequality, resistance to cultural integration, the conflict between conservative and modern worldviews, and the role of women in the transition towards cultural hybridity. The analytical and descriptive approach adopted in this study enables a comprehensive interpretation of both the novel and its female characters. This includes an examination of the influence of the characters, superstitious beliefs and practices, and the ways in which cultural heritage shapes the lives of women depicted in the narrative. In *Maps for Lost Lovers*, female characters function both as a critical lens through which social norms and injustices are examined and as a means of highlighting the restrictions imposed on individual freedoms as well as the impact of cultural and religious expectations.

Keywords: Nadeem Aslam; *Maps for Lost Lovers*; Pakistan; women; Mah-Jabin.

Nadeem Aslam's *Maps for Lost Lovers* portrays the postcolonial transitional period during which Pakistani and South Asian communities in Britain struggled to negotiate competing notions of nationhood and modernity. Like Aslam's other novels, this work is written in rich, poetic prose and has been widely praised by critics for its vivid imagery and emotional depth. The narrative is set in a fictional town modelled on Huddersfield in post-industrial northern England and revolves around a Pakistani family that migrated to Britain in the 1970s. Although their three adult children have become integrated into mainstream British society, they have, to varying degrees, grown increasingly detached from both their parents and their ethnic community.

The novel presents a number of significant female characters. Among Kaukab, Suraya, Chanda, Kiran, and numerous unnamed women whose lives are recounted throughout the narrative, Mah-Jabin stands out because of her distinctive worldview, personality, and way of life. As M. A. Hossain observes, “Pakistani migrants are unable to reconstruct their identities in a complete sense” (Hossain, 2020, p. 97). Mah-Jabin is one of the characters who actively attempts to reconstruct her identity and ultimately succeeds in achieving a greater degree of self-realisation.

Several of the novel's female characters are portrayed within the context of forbidden love and the chain of violent events that such relationships set in motion. Chanda is murdered by her brothers for falling in love with and living alongside Jugnu; a Muslim girl who has a Hindu lover is killed during a violent exorcism ritual organised by her own family; and Kiran's relationship with her beloved—the brother of the devout Kaukab—is forbidden because she is Sikh. The representation of these women serves the author's broader artistic purpose: “Although the various shocking incidents in Aslam's novel—violent exorcism, the murder of women, and abortions—are based on real events, the author himself deliberately presents this approach not as an attempt to 'deliver a message' but as an exploration of personal experience and consciousness. This demonstrates how difficult it is to separate authorial intention from literary effect” (Moore, 2009, p. 11).

Mah-Jabin, the daughter of Shamas and Kaukab—the central characters of the novel and members of a migrant family who relocated from Pakistan to Britain—is the only female character who successfully undergoes cultural transformation, comprehends the characteristics of both Eastern and Western civilizations, compares them critically, and ultimately forges her own path. Introduced into the narrative through the statement that “their daughter Mah-Jabin telephoned about once a month and visited once or twice a year” (Aslam, 2004, p. 49), she is portrayed primarily through the lens of her ideological and emotional conflicts with her mother. By juxtaposing the contrasting worldviews of mother and daughter, Aslam illustrates not only the tension between the older and younger generations and between conservative and integration-oriented women, but also the possibility for Pakistani, and more broadly South Asian, women to transcend the boundaries of cultural difference. Mah-Jabin is thus presented by Aslam as a model of female emancipation.

At the age of twenty-seven, Mah-Jabin lives and works in London. Like many British Pakistani women, she was sent to Pakistan at an early age to marry, making the initial stage of her life representative of the conventional experience of migrant women. However, after two years the marriage ended in divorce: “She had divorced the cousin with whom she had lived in the pale green house in Sohni Dharti, the man she had gone to Pakistan to marry when she was sixteen. Her decision to divorce her husband had filled her mother with both sorrow and anger” (Aslam, 2004, p. 125). Aslam's observation that “the two years of marriage seemed as alien to her as though they belonged to someone else's story” (Aslam, 2004, p. 125) serves two significant functions. First, it suggests that such life stories are common among women and therefore no longer regarded as exceptional. Second, it demonstrates that Mah-Jabin has liberated herself from traditional stereotypes and, having overcome not only their physical oppression but also their psychological trauma, has entered a new stage of intellectual and personal development.

The first points of conflict between Mah-Jabin and her mother Kaukab concern hairstyle and clothing. Asking, “Is this how white girls wear their hair these days?” (Aslam, 2004, p. 126), Kaukab interprets her daughter's decision to abandon “the effort of growing it for some eighteen years” (Aslam, 2004, p. 126)—simply because she “just wanted a change” (Aslam, 2004, p. 126)—as a symbolic severance from her religious and national roots. For Kaukab, another important indication of her daughter's estrangement from those roots is her forgetting Eid al-Fitr, a religious festival which, in her view, “should not need to be remembered” (Aslam, 2004, p. 134): “In Pakistan no one needs to be reminded when it is Eid or Ramadan, just as it would be impossible for people here to be unaware of Christmas” (Aslam, 2004, p. 135).

In raising her daughter, Kaukab, like many women in conservative families, sought to prepare Mah-Jabin to become a devoted wife: she was expected to master the art of cooking to please her future husband, dedicate sixteen to twenty years of her life to him, avoid excessive contact with Western society, and remain faithful to religious traditions and cultural customs. However, Aslam demonstrates that the surrounding social environment possesses a powerful formative influence. Mah-Jabin is shaped not only by her family but, to an even greater extent, by the society in which she grows up. With her conservative religious outlook and profound sense of alienation from Western culture, Kaukab herself acknowledges that young women raised in Britain and sent to Pakistan to marry close relatives “were still influenced by everything they had absorbed from school, their teachers, their friends, and life in this country” (Aslam, 2004, p. 133), which prevented them from adapting successfully to married life.

Whereas Mah-Jabin adopts a rational outlook and plans her future accordingly, Kaukab withdraws into loneliness, passivity, and pessimism, sustained only by “the hope of reliving the past.” According to Aslam, most people live in the past because remembering is easier than thinking (Aslam, 2004, p. 361). Kaukab's fixation on the past gradually alienates her from the surrounding world and even contributes to her physical illness as negative thoughts consume her: “When the mind rejected a thought and the body agreed with it, spasms began in her stomach as though she had swallowed a poisonous substance that needed to be vomited out” (Aslam, 2004, p. 38).

Mah-Jabin has transcended the boundaries of the migrant enclave that Pakistani and other South Asian immigrants in Britain call “Dasht-e-Tanhai.” Consequently, she rejects many of the assumptions held by members of her own community. Aslam explores these ideological differences primarily through the experiences of women. For example, Kaukab and a neighbouring woman interpret an incident in which an American police officer orders a Muslim girl to uncover her face as a violation of the unmarried girl's honour, while another woman claims that evil spirits have entered her daughter's body because “she refused to behave properly towards her husband" (Aslam, 2004, p. 146). Mah-Jabin regards such beliefs as profoundly primitive and irrational.

Just as Kaukab disapproves of her daughter's present way of life, she also opposes her future aspirations: “I forbid you to go to America.' She clenched her fist. 'A foreign country full of foreigners!” (Aslam, 2004, p. 149). Mah-Jabin, in contrast, sees her mother as “a woman imprisoned in a cage within which she was permitted to think” (Aslam, 2004, p. 149). Yet this metaphorical cage has not been constructed by any single individual. Rather, it has been created through Kaukab's upbringing in a deeply religious family that failed to develop a genuine understanding of Islam, her fear of integrating into a new social environment, her confinement within a rigid conception of religious and national identity, and her persistent resistance to cultural change. These factors have transformed her into “the most dangerous creature she would ever have to confront in life” (Aslam, 2004, p. 149). Mah-Jabin, however, consciously rejects her mother's path. Even her two-year marriage in Pakistan ultimately strengthens her determination to make independent choices: “I have lived in a country full of people like me and seen what it is like, so this time I decided to try a foreign country full of strangers” (Aslam, 2004, p. 149).

Aslam draws particular attention to women's freedom of choice, presenting Mah-Jabin as a woman who is capable of learning from experience and remaining committed to the decisions she makes. It was, in fact, her own decision to go to Pakistan for marriage. However, as she matures, she comes to realise that the appropriateness of personal choices and responsibilities depends largely on one's age and level of maturity: “I was sixteen. However you look at it, I was a child in your eyes. Then why did you leave a decision that should have been made by a mature adult to me? Other parents would have given me time to think, but you were overjoyed simply because I wanted to live in your beloved country” (Aslam, 2004, p. 150).

The character of Mah-Jabin is not created merely to illustrate her own moral and intellectual development. Through her, Aslam also seeks to uncover the limitations that have shaped Kaukab's life. Mah-Jabin plays a crucial role in revealing the more vulnerable dimensions of her mother's personality. Having observed Kaukab attentively since childhood, she reflects throughout the past nine years, as well as during the two years of her own marriage, on a single persistent question. According to Aslam, finding the right question can be just as difficult as discovering the right answer (Aslam, 2004, p. 151). Mah-Jabin recalls an occasion when, after listening to the opera singer Jessye Norman—whose language she did not even understand—Kaukab remarked, “I admire people who are capable of great things” (Aslam, 2004, p. 152). Kaukab had once confided to her daughter that she regretted never having had the opportunity to pursue an education and that, as a child, she had dreamed of owning a bicycle. However, her mother had refused, fearing that if she fell, injured herself, and became physically disabled, no one would ever wish to marry her. Although Kaukab understands deep within herself that such restrictions represented a profound injustice, she nevertheless attempts to suppress her daughter's aspirations in much the same way. Mah-Jabin therefore criticises her mother for her lack of courage, her inability to think rationally, her tendency to suppress truths even when she recognises them, and her unwillingness to embrace change.

Mah-Jabin is the only female character in the novel who undergoes continuous personal development. Although Chanda, one of the courageous protagonists of the Chanda–Jugnu narrative that occupies a central place in the novel's plot, defies social conventions, she remains a tragic figure whose story ends in death. Mah-Jabin, by contrast, survives. She continues to develop intellectually, exercises critical judgement, reflects upon the thoughts and behaviour of women, and remains open to new possibilities. She attributes the responsibility for Chanda's murder by her brothers, following her decision to live with Jugnu outside marriage, to those who share her mother's worldview: “You would tear her body apart, stitch the pieces together, and then kill her again simply because she challenged your rules, your laws, and those 'traditions' that you dragged into this country like filth clinging to your feet” (Aslam, 2004, p. 154). Here, Aslam reframes Chanda's murder as the consequence of collective responsibility and a culture of silence. The novel consistently emphasises the necessity of breaking that silence: “Shame, guilt, honour and fear are like locks hanging from people's mouths... this place is filled with hidden troubles” (Aslam, 2004, p. 45).

Mah-Jabin holds her mother responsible not only for the tragic fate of women such as Chanda but, above all, for Kaukab's own fragmented, isolated, and alienated identity. Kaukab is portrayed as a woman who suppresses the potential within herself. When she tells her daughter that “not everyone has the freedom to become childlike beyond a certain stage of life” (Aslam, 2004, p. 154), Mah-Jabin responds, “The fact that you could do that so easily made you arrogant and heartless” (Aslam, 2004, p. 154). She accuses her mother of denying her the freedom that she herself had never possessed and of failing to guide her away from making irreversible mistakes. Even Kaukab's admission—“I did not have the freedom to give you that freedom” (Aslam, 2004, p. 155)—fails to convince Mah-Jabin. In her view, Kaukab remains imprisoned by the same conservative and deeply entrenched stereotypes that continue to shape her identity. She is unwilling to break the chains that confine her.

Mah-Jabin has herself experienced many of the realities that shape the lives of Pakistani girls. “Mah-Jabin had been taught to be patient, because patience would eventually bring salvation” (Aslam, 2004, p. 158). Her family's decision to marry her to her Pakistani cousin was intended to rescue both young people from their respective “white” lovers. Shamas opposes the use of physical violence against his daughter, yet his motives extend beyond paternal concern: “Shamas had warned Kaukab not even to slap the girl, because otherwise the local newspapers would appear the next day with headlines such as 'Daughter of Pakistani Muslim Community Leader Beaten over Marriage Issue,' and the reputation of Islam and of the Pakistani migrant community—both in Britain and in the workplaces officially allocated to them in the name of justice—would immediately be damaged” (Aslam, 2004, p. 160). Aslam thus suggests that public reputation often becomes more significant than the emotional well-being and individual rights of women.

After the death of the wife of Shamas's elder brother in Sohni Dharti, Shamas and Kaukab decided to relieve the widower, his son, and the household of its “womanlessness” (Aslam, 2004, p. 160). They therefore “asked their daughter whether she wished to marry her cousin, and the girl answered, 'Yes'” (Aslam, 2004, p. 160). Here, Aslam exposes the deeply flawed logic underlying such marriages and the reduction of women to domestic caregivers whose primary function is to provide service within the family. The consent of a sixteen-year-old girl—whose agreement reflected her age, upbringing, and emotional immaturity—was regarded as sufficient to determine the course of her entire life. As the narrator observes, “It suited everyone, and when the girl's silent dreams over the previous two years—silent, boundless, misguided, innocent, and uncontrolled dreams—had begun to imagine a different future for her destiny, Kaukab saw in them proof that Allah could blind one's judgement” (Aslam, 2004, p. 161). Consequently, after her divorce, and following the profound transformation of her outlook on life and her moral and psychological development, Mah-Jabin holds her mother responsible for failing to intervene and prevent the life-altering decision made by her sixteen-year-old daughter.

In the final chapter of the novel, “Autumn,” Aslam once again places particular emphasis on the character of Mah-Jabin. Like Kaukab's other children, she returns to attend the trial of Jugnu's and Chanda's murderers. In this section, Mah-Jabin attempts to protect her mother from the anger and resentment of her brother Ujala. Having completely distanced himself from traditional Pakistani customs, Ujala has never forgotten that his mother once gave him a poisonous substance. Although Kaukab believed she was giving him “salt over which the imam had recited sacred verses” in an attempt to prevent him from abandoning their customs, she remained unaware that the substance was, in fact, poisonous. Moore argues that Mah-Jabin also understands that Kaukab's actions stem from fear and misunderstanding (Moore, 2009, p. 7). This interpretation is persuasive. Although Mah-Jabin challenges her mother on many issues, she also recognises that Kaukab's actions are ultimately motivated by love for her children, however misguided their expression may be.

Mah-Jabin's suffering is further revealed through a letter written by her former husband, which Kaukab reads by chance. The words, “Do you remember the burn from my cigarette on your body, Mah-Jabin? Never forget that pain. The flames of Hell are a thousand times hotter” (Aslam, 2004, p. 389), horrify her mother. Through this letter, Aslam exposes the cruelty inflicted upon women by a man who regards himself as a devout believer: “Do you remember the sewing needles in your feet, Mah-Jabin?” (Aslam, 2004, p. 390).

Mah-Jabin's former husband, who considers a woman's foremost duty before God to be absolute obedience to her husband, accuses her of lacking faith. Through these revelations, Aslam creates the conditions for Kaukab to re-examine the truths upon which she has built her life: “Kaukab sat wondering whether this was indeed who she was, whether this was the reflection she saw in the mirror: was she a mother who had poisoned her son, who had passed judgement so hastily, and who had blamed her daughter for the collapse of her marriage?” (Aslam, 2004, p. 392). The novel thus invites readers to reflect upon a fundamental question: “You cannot know how easy it is to destroy a woman's life” (Aslam, 2004, p. 400). Unlike many of the novel's female characters, Mah-Jabin succeeds in escaping this destructive cycle. She divorces the husband who subjected her to abuse and finds the courage to continue her life under more dignified and fulfilling circumstances.

Mah-Jabin also plays a significant role in revealing the evolution of Kaukab's own thinking. “You brought me here, to this cursed country. You took my children away from me” (Aslam, 2004, p. 416), Kaukab tells Shamas, holding him responsible for the family's misfortunes. She blames him for her daughter's suffering at the hands of an abusive husband, for failing to move the family to a better neighbourhood where their children could have grown up under more favourable conditions, for not attaining a higher social position that might have enabled him to find a more suitable husband for their daughter, and even for failing to secure the prosperous future she had imagined despite their eldest son Charag's becoming a doctor, believing that having a physician as a brother would improve her daughter's marriage prospects.

Like Aslam's other female characters, Mah-Jabin is also significant for understanding the relationship between British society and immigrant communities. Her decision to travel to Pakistan at the age of sixteen and marry her cousin was shaped, in part, by the constraints she experienced within the Western social environment. Drawing attention to the circumstances faced by migrant women, P. Ravindran argues that “they strive to maintain a balance between adapting to the local culture and preserving their religious values. Discrimination, stereotyping, and Islamophobia remain among the principal challenges they encounter” (Ravindran, 2023). In this context, Moore observes that Aslam's novel does not simply promote the ideal of “loving one's neighbour.” Rather, it demonstrates that multicultural policies have failed to resolve problems within immigrant communities, such as forced marriages. From this perspective, “the novel simultaneously exposes both external stereotypes and internal social tensions” (Moore, 2009, p. 10).

As is evident, although Mah-Jabin is not presented as one of the novel's most prominent characters, she plays a central role in articulating several of its most significant themes. Aslam effectively employs Mah-Jabin to reveal the hidden thoughts, aspirations, and emotional conflicts of Kaukab, one of the novel's principal protagonists. At the same time, Mah-Jabin serves as an important medium through which family relationships, the experiences of her siblings, her mother's worldview, and broader perceptions of Pakistan are explored. Most importantly, through the character of Mah-Jabin, Aslam constructs the portrait of a Pakistani woman who, despite enduring the pressures imposed by her family, society, cultural environment, and deeply rooted stereotypes from an early age, ultimately finds the strength to overcome oppression and hardship. She emerges as a woman who possesses the courage to pursue her aspirations and remain faithful to the values and convictions she believes to be right.

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THE EMBODIMENT OF THE NIZAMI TRADITION IN TURKIC LITERATURE: QUTB'S KHOSROW AND SHIRIN MASNAVI

Musayeva A.
Baku State University (Master's Student)
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Abstract

Nizami Ganjavi's *Khosrow and Shirin* is one of the most influential works of Eastern literature and one of the subjects that has been repeatedly reinterpreted in different languages throughout subsequent centuries. This work left a profound impact not only on the Persian literary tradition in which it was created but also on Azerbaijani and other Turkic literatures, contributing to the formation of a rich literary tradition. This article examines the historical origins of the *Khosrow and Shirin* narrative, the artistic and aesthetic innovations introduced by Nizami, and the poem's place in Eastern literature. Particular attention is devoted to the masnavi of the same title by the Qizil Orda poet Qutb, one of the earliest and most significant continuators of the Nizami tradition in Turkic literature.

Qutb's *Khosrow and Shirin* was composed on the basis of Nizami's poem; however, it goes beyond the framework of a simple translation and acquires distinctive artistic characteristics. The article summarizes scholarly views concerning the linguistic features of the work, its surviving manuscript, publication history, and academic studies. It concludes that Qutb's *Khosrow and Shirin* is a valuable literary monument that played a significant role in disseminating Nizami's legacy throughout the Turkic-speaking world, preserving the Nizami tradition, and contributing to the development of the Turkic literary language.

Keywords: Nizami Ganjavi, *Khosrow and Shirin*, Qutb, Qizil Orda literature, medieval literary language.

Introduction. In the twelfth century, the poetry of the Azerbaijani poet Nizami Ganjavi emerged as a unique and invaluable phenomenon for the entire literary tradition of the East. Over the centuries, his works spread across a vast geographical area and diverse national and cultural environments, exerting a profound influence on the development of artistic thought. The rich literary heritage of this great poet, which has become an inseparable part of the spiritual world of the Azerbaijani people, has maintained its distinguished and enduring place within the treasury of Eastern cultural heritage for centuries. It may be said that in the literatures of many Eastern peoples, prominent representatives of literary schools founded upon the Nizami tradition emerged, and their creative activity contributed both to the enrichment of national literatures and to their establishment within the history of world literature. This process constituted one of the interconnected and parallel directions of long-term literary development. The other direction involved the preservation, study, and broad dissemination of the ideological and intellectual legacy of the great Azerbaijani poet.

One of the most important manifestations of this legacy is the reinterpretation of Nizami's subjects and plots within the literary traditions of different peoples and their artistic expression in various languages. In this respect, one of the most significant themes is the *Khosrow and Shirin* narrative, which became widely known throughout Eastern literature. Over time, this narrative was not confined to Persian literature alone but also entered the Turkic literary tradition, where it acquired new poetic interpretations.

The historical roots of the *Khosrow and Shirin* story predate Nizami. According to M. Tarbiyat, the first independent literary version of this story was composed by the renowned twelfth-century Iranian poet Sana'i. Nevertheless, certain motifs and episodes of the narrative had appeared earlier in prose works such as al-Jahiz's *Al-Mahasin wa al-Aḍdad*, Abu Mansur al-Tha'alibi's *Ghurar al-Akhbar*, Abu Miskawayh's *Nadim al-Farid*, and Ibn Nubata's *Sharh al-'Uyun*. In addition, Ferdowsi narrated several episodes of the legend from a historical perspective in his *Shahnameh* [7, p. 257]. While Ferdowsi's *Shahnameh* provides a detailed account of Khosrow's political struggles and briefly mentions his love story with Shirin, it was Nizami Ganjavi who, for the first time, transformed the legend into verse, thereby granting it literary immortality. Following Nizami, the subject was revisited by numerous poets and thinkers in different periods, each approaching it with distinctive artistic interpretations, thus making a substantial contribution to the shared literary and cultural heritage of the East.

In this context, one of the most important continuations of the Nizami tradition in Turkic literature is Qutb's *Khosrow and Shirin*. This work deserves particular scholarly attention as a significant literary monument that reflects the creative adaptation and new poetic expression of Nizami's narrative within the Turkic literary milieu.

Main Part. Nizami Ganjavi, the outstanding representative of twelfth-century Azerbaijani literature and a distinguished poet-philosopher, is regarded not only as one of the greatest literary figures of Azerbaijan but also as one of the most eminent poets in world literature. Through his rich literary heritage, he made a significant contribution to the development of Azerbaijani written literature and exerted a profound influence on the literature and culture of the peoples of the East. Nizami began his literary career by composing lyric poetry in such genres as the ghazal, qasida, and rubai. However, he achieved worldwide recognition primarily through his Khamsa ("Quintet"), consisting of the five masnavis The Treasury of Mysteries, Khosrow and Shirin, Layla and Majnun, The Seven Beauties, and The Book of Alexander.

The second poem of Nizami Ganjavi's *Khamsa*, *Khosrow and Shirin*, is considered one of the finest masterpieces of both Azerbaijani and world literature. Through this work, Nizami laid the foundation of the romantic narrative poem in the literature of the Near East and paved the way for the emergence of a new literary movement-the Nizami literary school. The poem, consisting of 6966 couplets, was completed in 1181 and dedicated to Shams al-Din Jahan Pahlavan. Its plot is based on the widely known Eastern legend of the Sasanian ruler Khosrow Parviz and the Azerbaijani beauty Shirin. Distinguished by its profound content, elevated poetic language, original composition, and rich gallery of characters, the poem has remained the focus of readers and scholars for centuries. As stated, “This romantic poem presents love as a great force capable of purifying human beings from base sensual desires and elevating them to self-sacrifice for the sake of the beloved” [5, p. 154]. This idea constitutes the principal artistic and aesthetic value of the work and is one of the main reasons for its enduring relevance.

The earliest literary reference to the story of *Khosrow and Shirin* appears in Ferdowsi’s *Shahnameh*. In his account, Ferdowsi primarily describes Khosrow’s reign, portraying his royal splendor, military campaigns, and political achievements in the characteristic style of epic historiography. Nizami, by contrast, places love at the center of the narrative, emphasizing its purifying power, clarifying its decisive role in human destiny, and depicting the process of moral perfection through vivid characters endowed with profound psychological depth [6, p. 7].

There is no doubt that Nizami Ganjavi breathed new life into this legend and elevated it to the rank of world literary classics. Through the pen of the Turkic poet Nizami Ganjavi, the story of *Khosrow and Shirin* reached its highest artistic expression in Persian literature. After Nizami, the subject was revisited by numerous poets in Persian, Indian, and Turkic literatures, who rewrote it with various additions, modifications, and abridgements. Hamid Arasli observed that the influence of the great master developed in two directions. His followers either adopted his plots in their entirety and created completely new literary works, or they followed Nizami’s innovative artistic method, adopting the structural principles of his poems while producing independent compositions [5, p. 154]. Thus, the literary tradition established by Nizami continued to flourish and exerted a lasting influence on the development of Eastern literature.

For centuries after Nizami, nearly thirty Persian poets, including Amir Khusraw Dehlavi, Vahshi Bafqi, Hatifi, and Urfi Shirazi, turned to this subject.

F. K. Timurtaş notes that after Nizami, eighteen Turkic poets composed works on this theme [9, p. 70]. The first among them was Qutb, a poet of the Qizil Orda. Through his translation of Nizami’s work, the story entered the Turkic literary tradition and later reached its finest artistic expression in the works of poets such as Sheikhi and Alisher Navoi.

The masnavi *Khosrow and Shirin*, produced within the literary milieu of the Qizil Orda, was composed by Qutb between 1341 and 1342. He translated Nizami’s Persian masnavi of the same title into Turkic and dedicated it to Tini Beg Khan, the ruler of the Qizil Orda, and his wife, Malika Khatun.

To date, reliable information regarding Qutb’s life and identity remains scarce. According to Aysu Ata, J. Eckmann suggested that he originated from the Khorezm and Mawarannahr region. F. Köprülü, relying on the linguistic features of the work, attempted to determine the literary and cultural environment to which Qutb belonged. Noting the strong influence of Khaqani Turkic in the text, he concluded that the poet most likely came from Transoxiana [1, p. 49].

Ü. Ö. Demirci states that among all Turkic versions of *Khosrow and Shirin*, Qutb’s masnavi is the most accomplished. While translating the work from Persian, Qutb did not adhere rigidly to Nizami’s original text. Instead, he omitted certain passages, modified others, and, in some instances, enriched the narrative with original couplets that departed from the source text. At the same time, in adapting the masnavi to the language of the Qizil Orda, he skillfully exploited the expressive potential of the Turkic language, incorporating numerous proverbs and sayings characteristic of the Turkic vernacular of his period. His replacement of approximately 64 percent of the Persian vocabulary in Nizami’s highly artistic masnavi with words of Turkic origin clearly demonstrates both his literary mastery and his profound command of the language of his age [2, p. 149]. Qutb’s approach indicates that he did not limit himself to a literal translation. While preserving the principal ideas and narrative structure of the original work, he introduced creative modifications in certain sections and adapted the text to the linguistic and aesthetic expectations of Turkic-speaking readers. These features distinguish Qutb’s *Khosrow and Shirin* from an ordinary translation and endow it with the character of an independent literary work.

According to the prominent Turkologist N. Hacıeminoğlu, Qutb remained faithful to Nizami with regard to the plot structure, the sequence of events, and the conclusion of the narrative. Nevertheless, whereas the extant version of Nizami’s masnavi comprises approximately 5700 couplets, the surviving manuscript of Qutb’s translation contains 4370 couplets, indicating a difference of nearly 1300 couplets between the original and the translation. Hacıeminoğlu also points out that Qutb preserved the metrical structure of the original, with the sole exception of the eulogy to the *Chahar Yar*, which he composed in the metre of *Kutadgu Bilig* [4, p. VII].

While translating the work, Qutb also incorporated certain elements reflecting the historical and cultural environment of his own period. Furthermore, the poem contains a number of artistic features that reveal his individual literary personality and creative originality. As Hacıeminoğlu notes, Qutb’s poetic talent is clearly demonstrated by the 271 original couplets that he added to the beginning of the work, which confirm that he produced a highly successful Turkic translation of Nizami’s masnavi [4, p. VIII].

The only surviving manuscript of the work is preserved in the National Library of France under manuscript number 312. The first scholarly description of this manuscript was provided in 1921 by the French orientalist Jean Deny [5, p. 154]. The manuscript was copied in 1383, forty-two years after the composition of the original work, in Alexandria by a Kipchak scribe named Birga ibn Barakiz ibn Qandu for Altın Boğa.

The preservation of this unique manuscript has considerably enhanced its historical and scholarly significance. It serves not only as an essential source for the study of Qutb's literary legacy but also as an important document for investigating the fourteenth-century Turkic literary language and the literary traditions that developed within the Qizil Orda. In this respect, *Khosrow and Shirin* is valuable not only as a literary monument but also as one of the principal written sources reflecting the historical development of the Turkic language and literature.

Certain metrical irregularities occur in some parts of the manuscript. However, it is difficult to believe that a poet possessing sufficient literary talent to translate Nizami's masterpiece into Turkic would have made such elementary metrical mistakes. The metrical defects found in the sixty-six couplets appended to the end of the manuscript by the copyist provide strong support for this view [4, p. VIII-IX].

The work, discovered by A. N. Samoylovich in 1913, was published in Warsaw in 1958 by the Polish scholar Professor A. Zajaczkowski in three volumes, including a transcription, a facsimile edition, and a glossary. N. Hacıeminoğlu later prepared the work as his doctoral dissertation and published it in 1968 together with its text, grammatical analysis, and index. Among the major scholarly studies devoted to the work are Faruk Kadri Timurtaş's article *Türk Edebiyatında "Husrev ü Şirin ve Ferhad u Şirin Hikâyesi"* (1959) and Abdülkadir İnan's article "*Kutbun Hüsrev ü Şirin'inden Örnekler*" (1951).

There are differing scholarly opinions regarding the variety of Turkic in which *Khosrow and Shirin* was written. According to Aysu Ata, the first scholar to examine the language of the work was A. N. Samoylovich, who identified it as a mixture of Uyghur and Kipchak Turkic [1, p. 50]. N. Hacıeminoğlu, on the other hand, argues that, from the perspective of verbal morphology, the language of *Khosrow and Shirin* belongs to the period preceding Chagatai Turkic, while its phonological features and vocabulary display characteristic traits of Kipchak Turkic [3, p. 13].

Qutb's translation of this work into Turkic in the fourteenth century represents a significant literary achievement. By rendering such an important romantic masnavi into Turkic, he demonstrated the rich expressive potential of the language of his time. At the same time, he introduced into the written literary language numerous words of Turkic origin that had previously survived only in the spoken vernacular, thereby contributing substantially to the enrichment of the literary vocabulary. Consequently, Qutb played a crucial role not only in transmitting Nizami's literary heritage to Turkic-speaking readers but also in the development and enrichment of the Turkic literary language.

Conclusion and Scientific novelty. The present study demonstrates that Nizami Ganjavi's Khosrow and Shirin laid the foundation for one of the most influential literary traditions in Eastern literature. One of the earliest and most successful representatives of this tradition in Turkic literature was Qutb. While preserving the original plot and ideological content of Nizami's poem, he creatively reworked the text within the linguistic framework of Qizil Orda Turkic and enriched it with distinctive artistic features. Consequently, Qutb's Khosrow and Shirin may be regarded as a valuable literary monument of great importance both for preserving the Nizami tradition and for contributing to the development of the Turkic literary language.

The scientific novelty of this article lies in its comprehensive analysis of the linguistic, thematic, and artistic characteristics of Qutb's *Khosrow and Shirin*, one of the earliest and most significant Turkic translations and adaptations of Nizami Ganjavi's poem. The study evaluates Qutb's work not merely as a translation but as an independent literary composition creatively reinterpreted from the original. Furthermore, it summarizes the poet's contribution to the dissemination of Nizami's literary heritage and to the historical development of the Turkic literary language.

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